

The phase diagram of the Higgs $SU(2)$ model

C. Bonati, G. Cossu, A. D'Alessandro,
M. D'Elia, A. Di Giacomo

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Convegno Informale di Fisica Teorica



Summary

- the model (notations)



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- features of the model



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- previous results in literature



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- features of the model
- previous results in literature
- new results
- conclusions

The model

$SU(2)$ gauge theory coupled with an Higgs in fundamental representation

$$S = S_W[U] - \frac{\kappa}{2} \sum_{x,\mu} \{ \Phi^\dagger(x) U_\mu(x) \Phi(x + \hat{\mu}) + \Phi^\dagger(x + \hat{\mu}) U_\mu^\dagger(x) \Phi(x) \} + \lambda \sum_x [\Phi^\dagger(x) \Phi(x) - 1]^2$$

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in this work $\lambda = \infty$

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$$\tilde{\Phi}(x) = i\sigma_2 \Phi(x)^* \quad \phi(x) = \begin{pmatrix} \tilde{\Phi}_1(x) & \Phi_1(x) \\ \tilde{\Phi}_2(x) & \Phi_2(x) \end{pmatrix}$$
$$\Phi^\Lambda = \Lambda \Phi \quad \longrightarrow \quad \phi^\Lambda = \Lambda \phi$$

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$$S = \beta \sum_{x,\mu < \nu} \left\{ 1 - \frac{1}{2} \text{ReTr} P_{\mu\nu}(x) \right\} - \\ - \frac{\kappa}{2} \sum_{x,\mu} \text{Tr} [\phi^\dagger(x) U_\mu(x) \phi(x + \hat{\mu})]$$

Features of the model

$$S = S_W[U] - \frac{\kappa}{2} \sum_{x,\mu} \text{Tr}[\phi_x^\dagger U_{x,\mu} \phi_{x+\mu}]$$

Limiting cases:

■ $\kappa = 0$ \longrightarrow $SU(2)$ gauge theory
confinement

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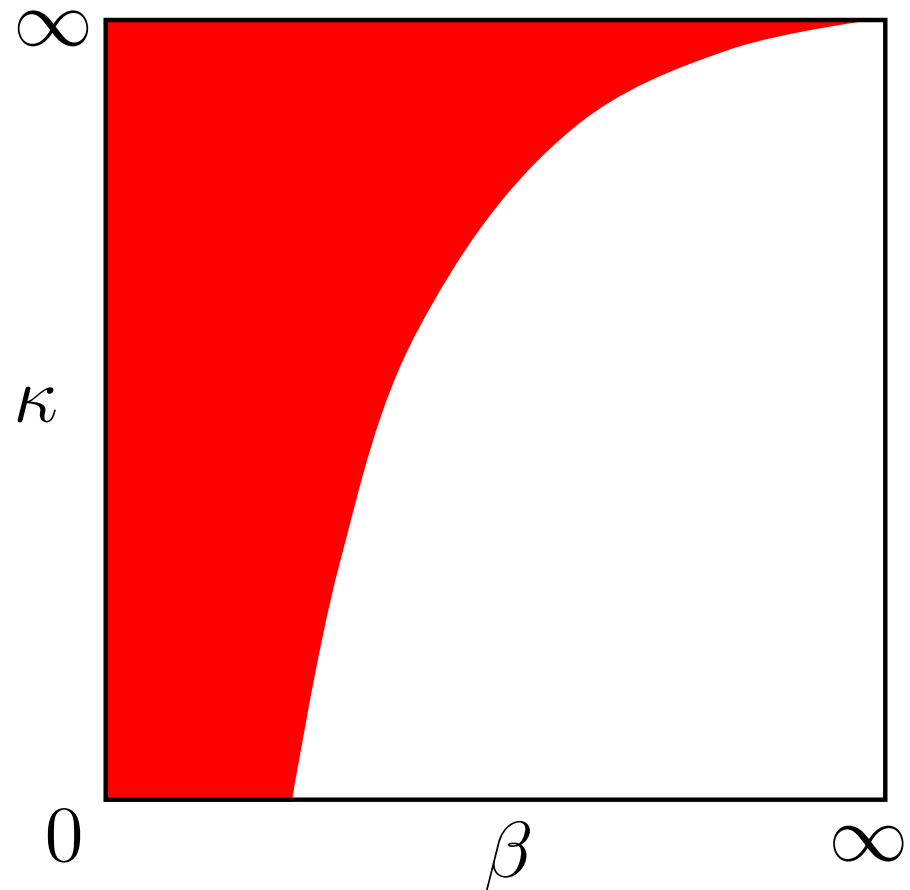
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- $\beta = 0$ & unitary gauge \longrightarrow independent $U_\mu(x)$'s
- $\kappa = \infty$ & unitary gauge \longrightarrow no dynamics

Features of the model

Fradkin-Shenker theorem:

Phys. Rev. D **19**, 3682 (1979)

in the **red region**
local observables
are analytic



Features of the model

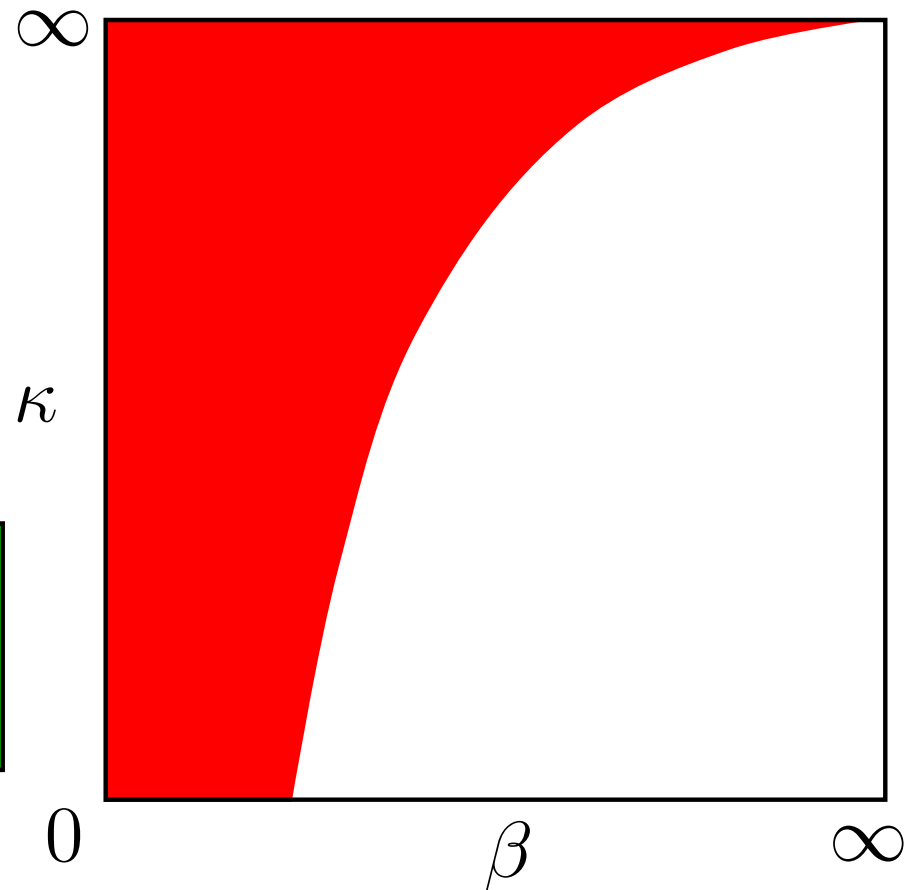
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The theorem does not hold
for *non-local* observables!

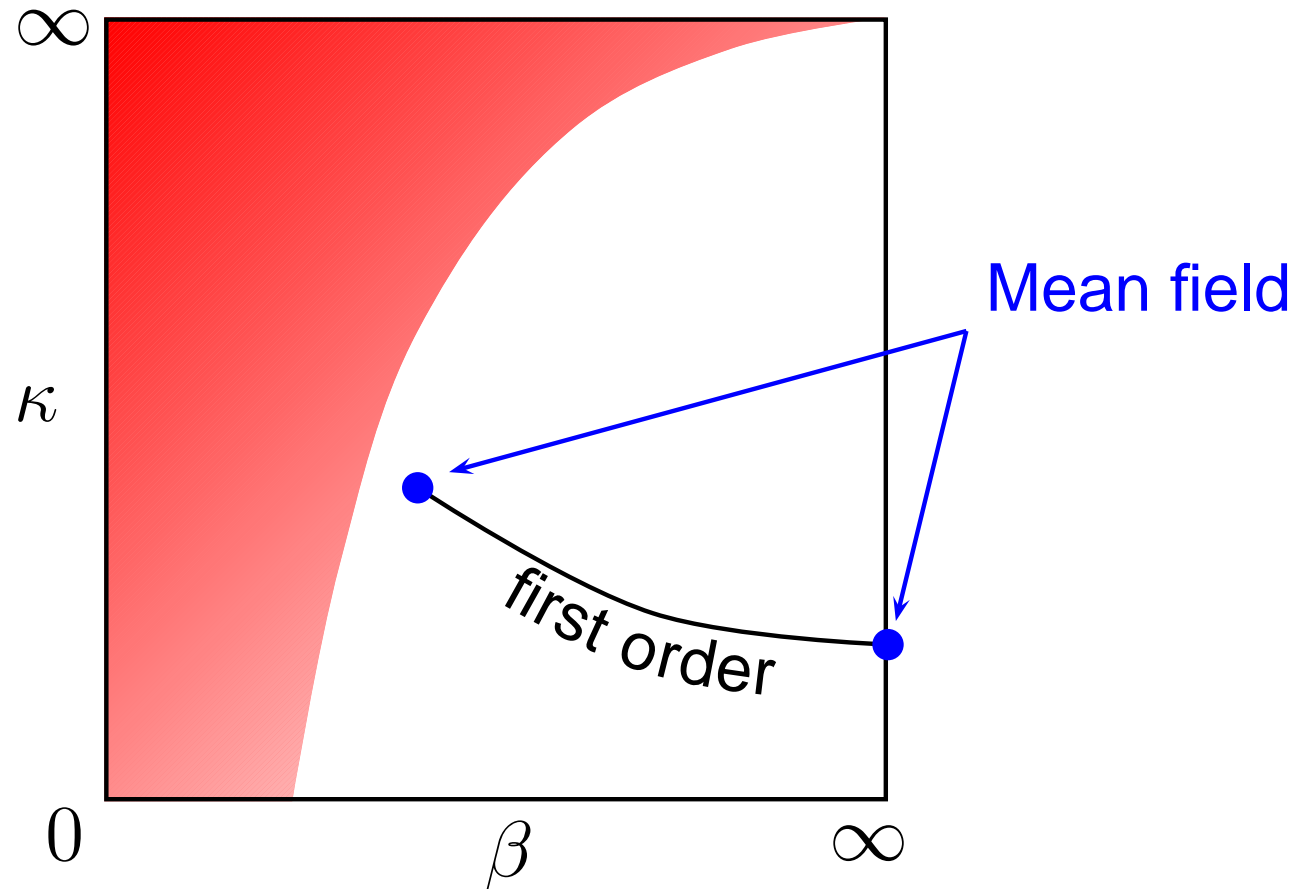
Grady, Phys. Lett. B **626**, 161 (2005)



Features of the model

Supposed phase diagram at zero temperature

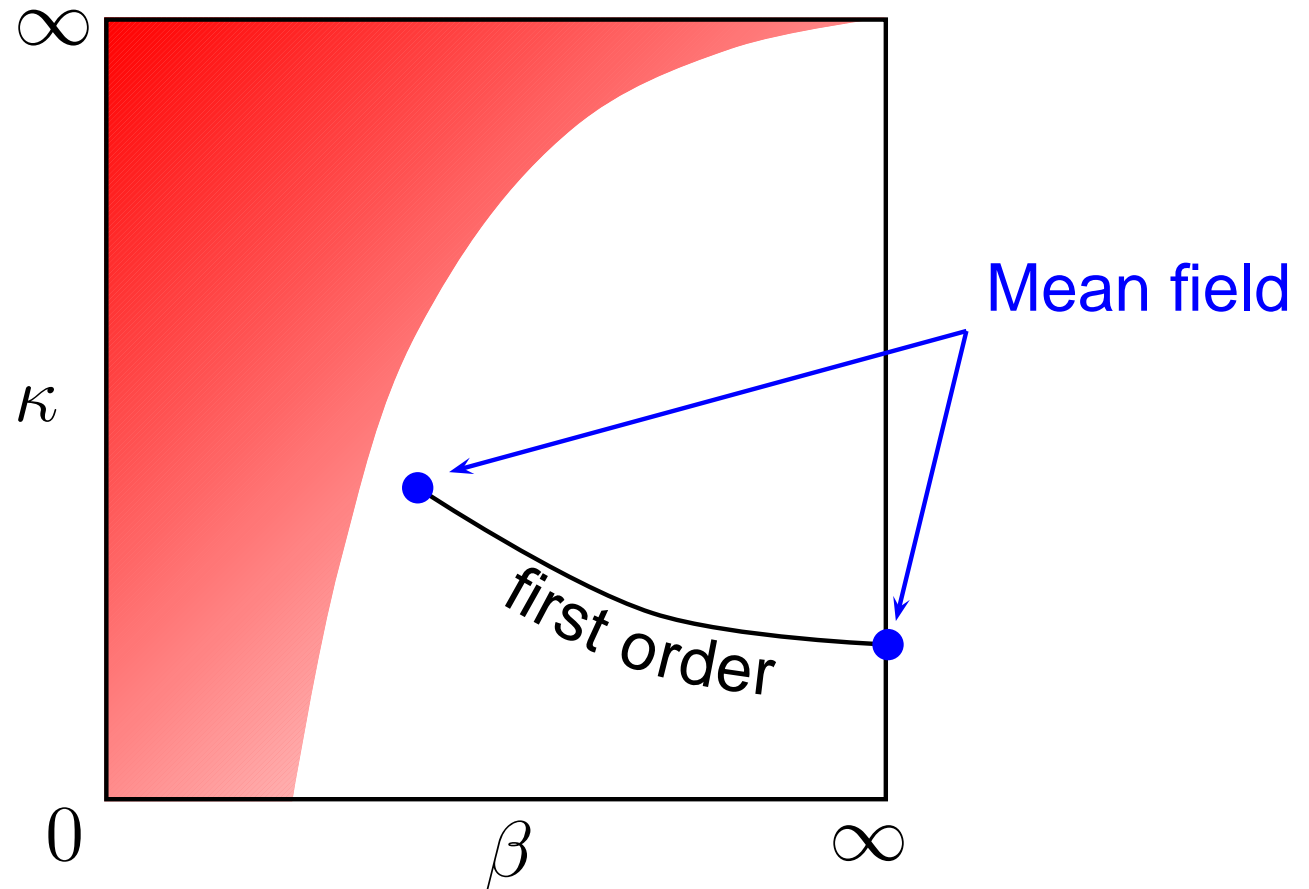
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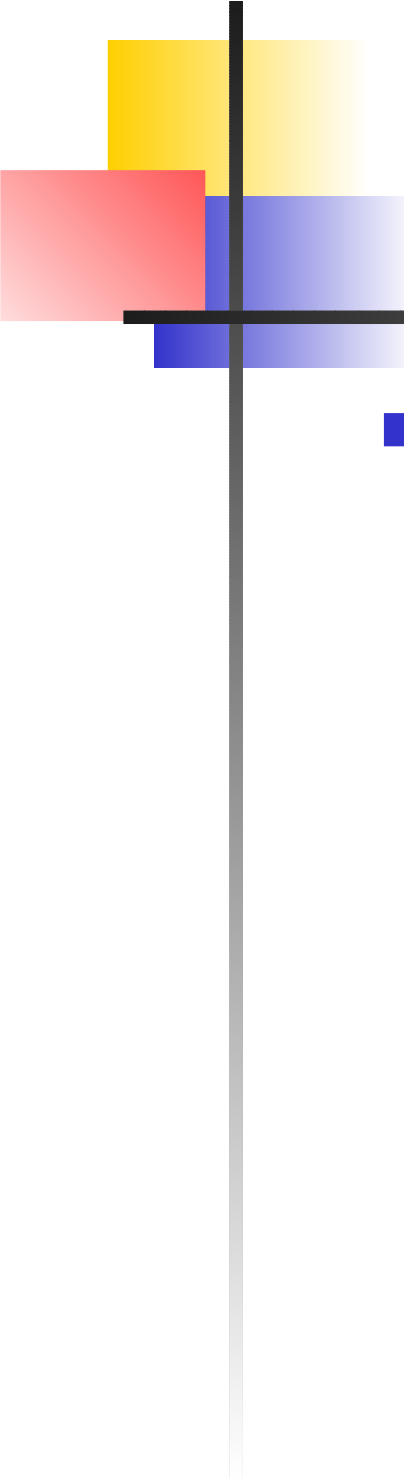
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Confirmed in
simulations at
 $\lambda = 0.5$

Bock et al. Phys. Rev. D
41, 2573 (1990)



Results in literature for the $\lambda = \infty$ case

- first numerical study on 4^4 lattice seems to confirm theoretical prediction

Lang, Rebbi & Virasoro Phys. Lett. B **104**, 294 (1981)

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Langguth & Montvay Phys. Lett. B **165**, 135 (1985)

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- “the system exhibits a transient behavior up to $L = 24$ along which the order of the transition cannot be discerned” (also in this case $\beta = 2.3$)

Campos Nucl. Phys. B **514**, 336 (1998)

New results

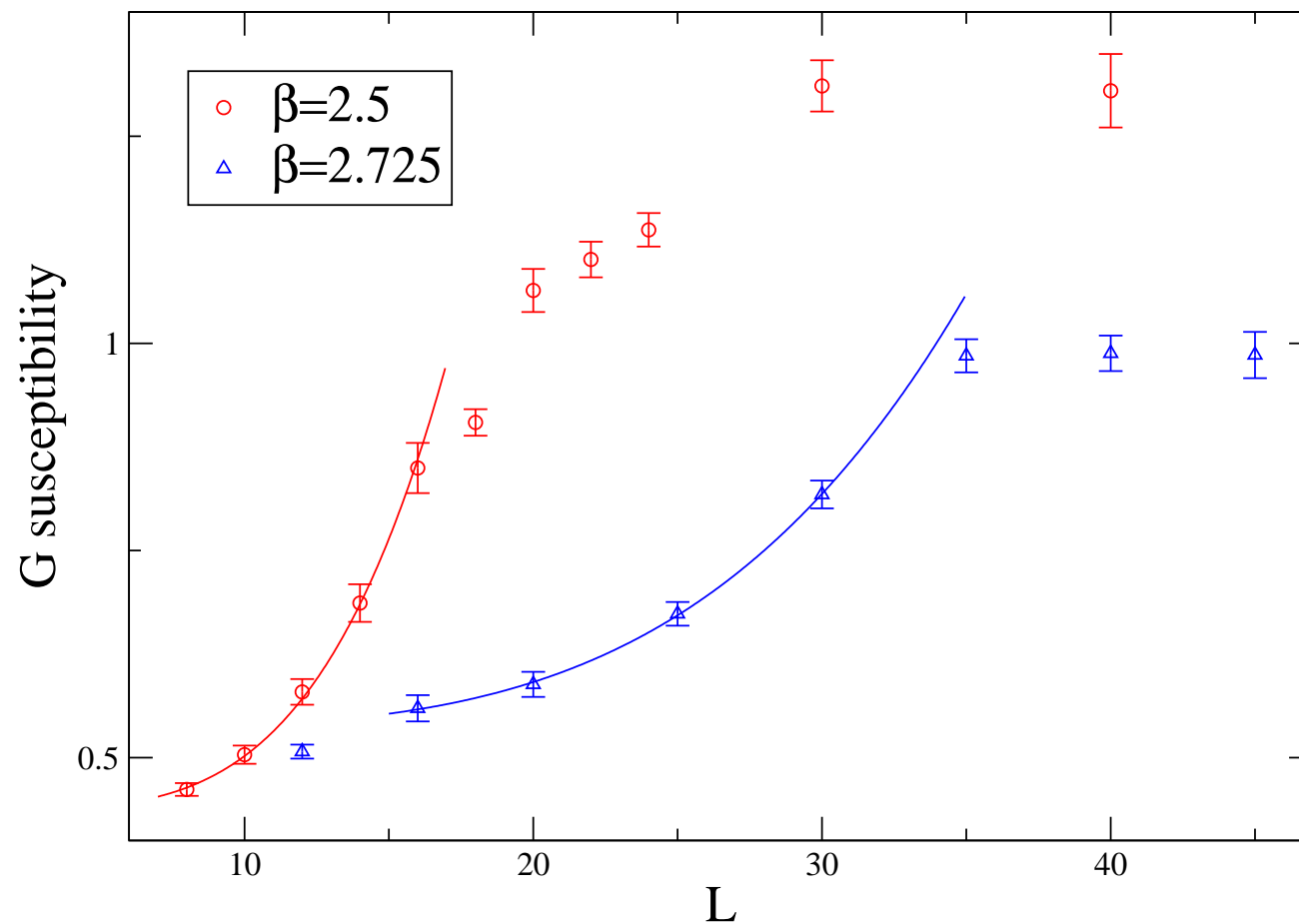
Local observables analyzed

- plaquette (P)
- Higgs-gauge interaction:
$$G = \frac{1}{2} \text{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x + \hat{\mu})]$$
- energy density $E = 6\beta P + 4\kappa G$
- Z_2 monopoles density (M)

Non-local observables analyzed

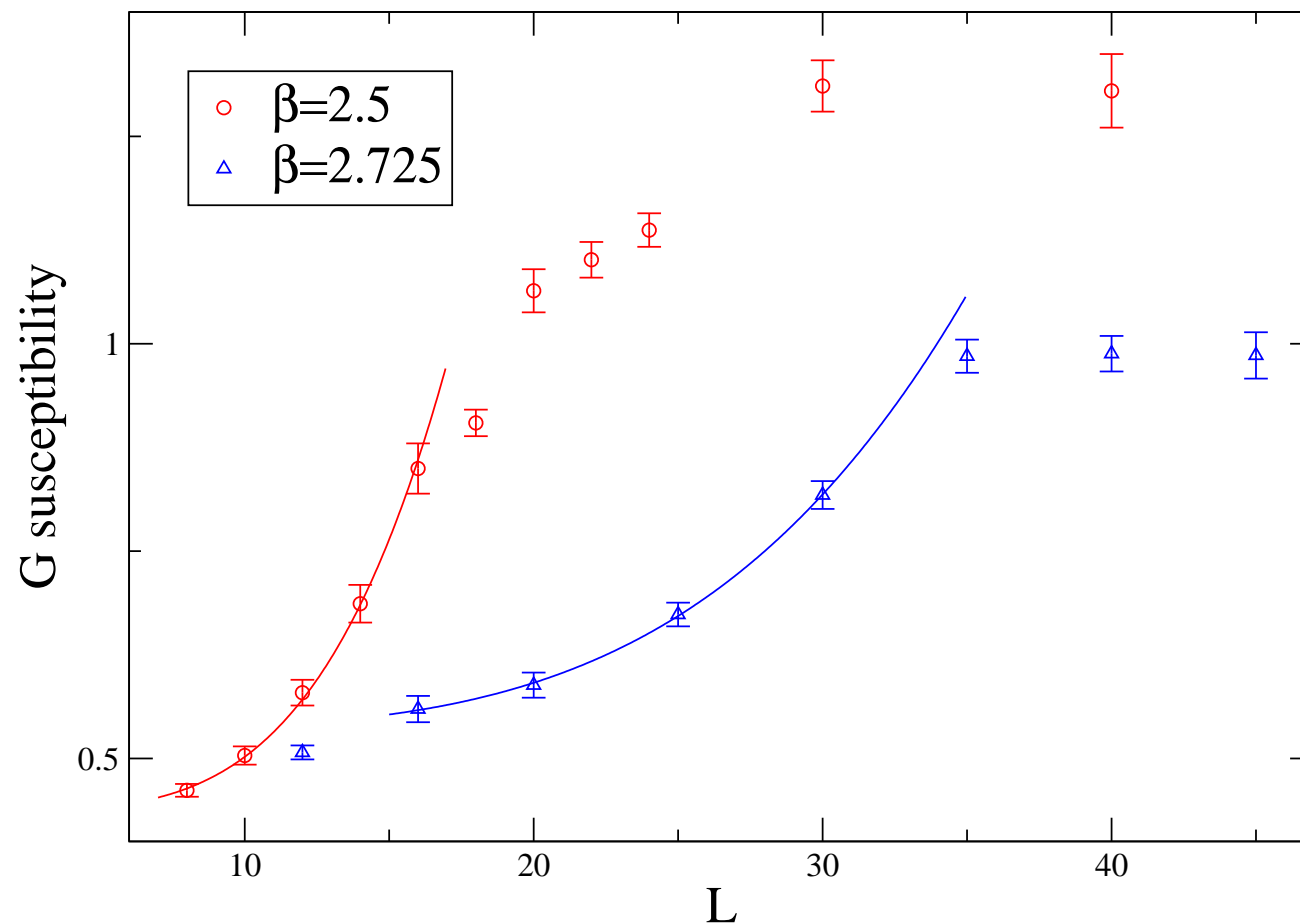
- Polyakov loop (P_L)

Results for “small β ” and $T = 0$



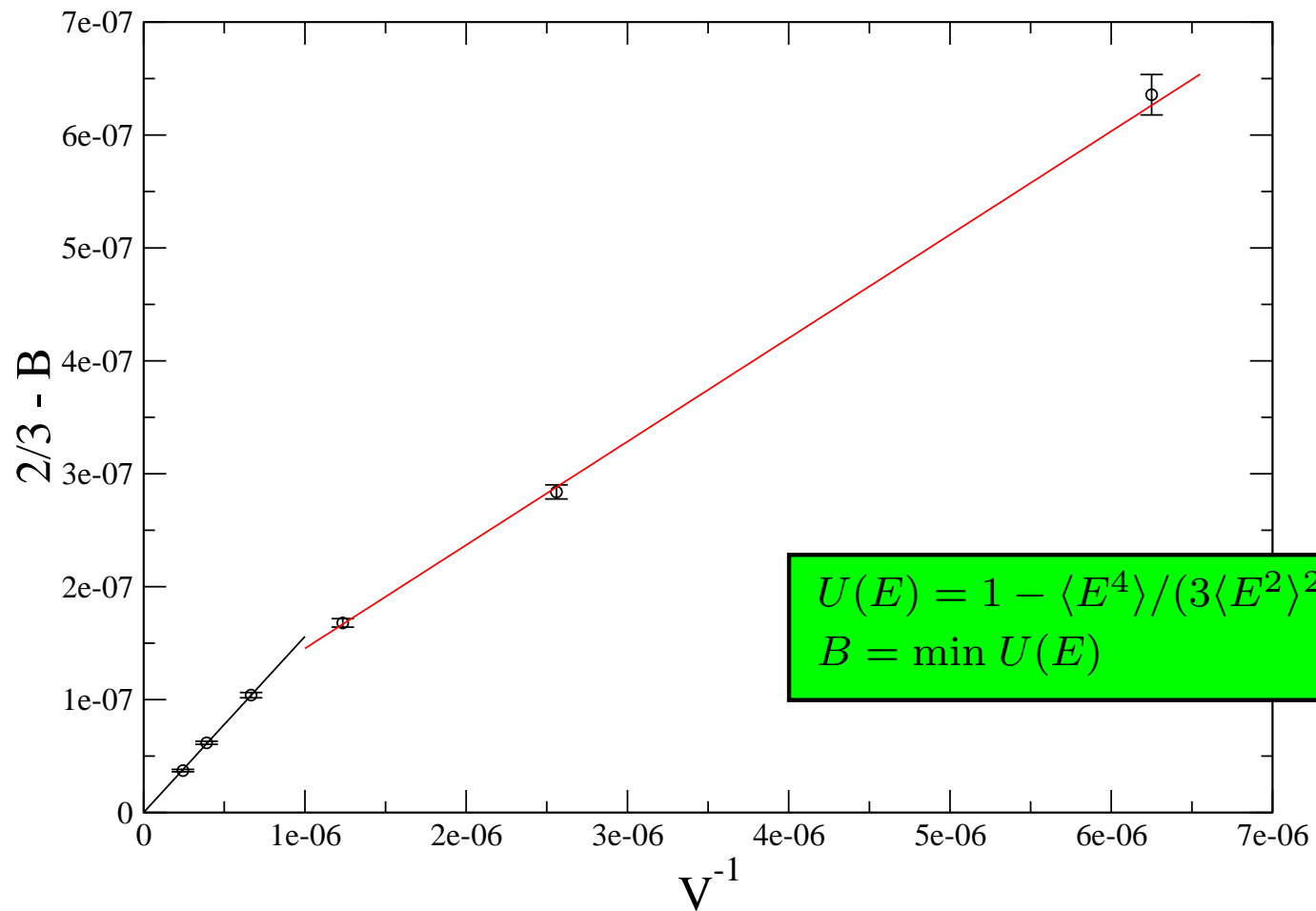
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Similar results for M , P and E susceptibilities



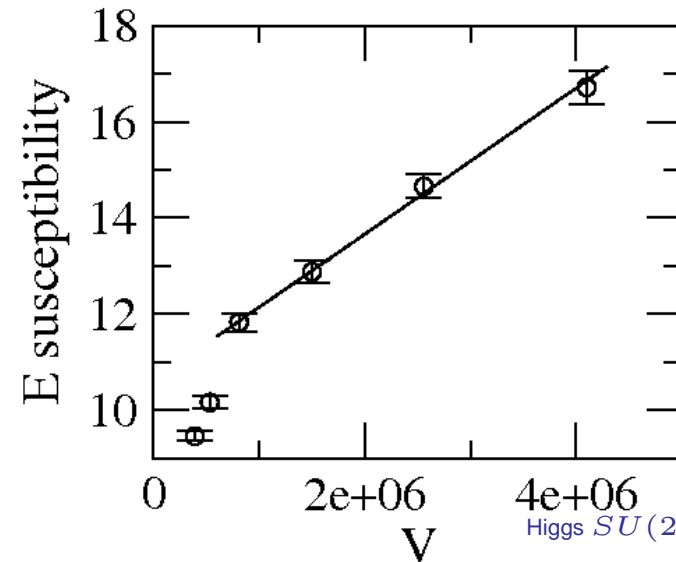
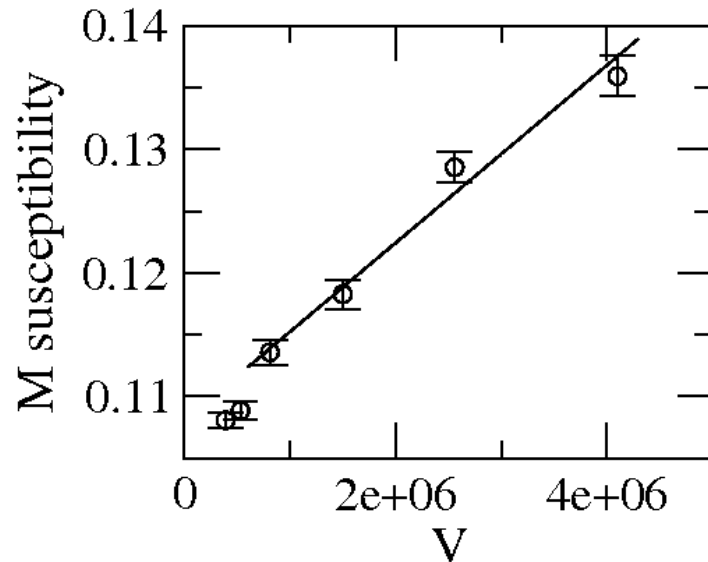
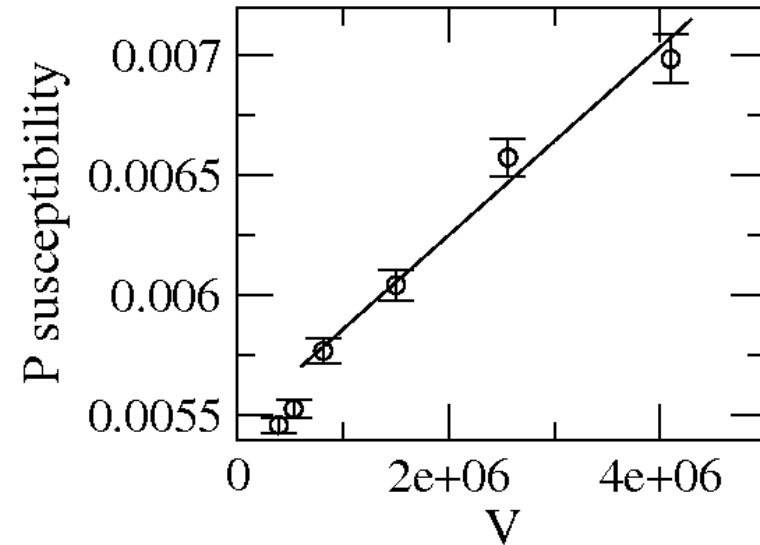
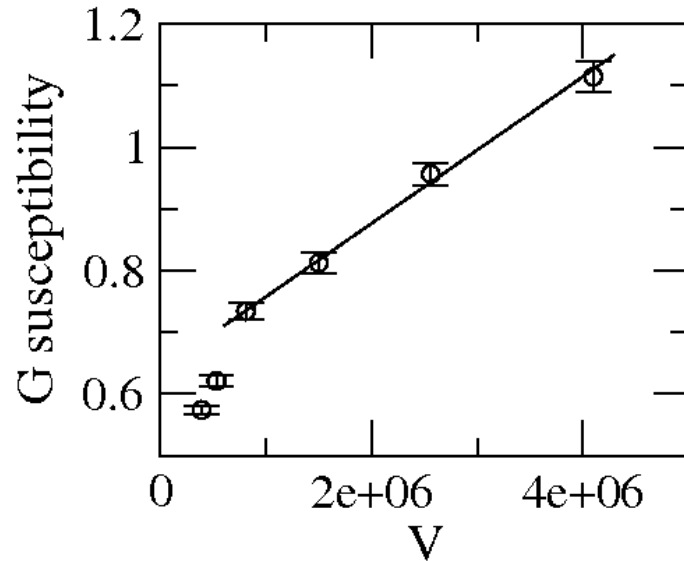
Results for “small β ” and $T = 0$

$\beta=2.725$

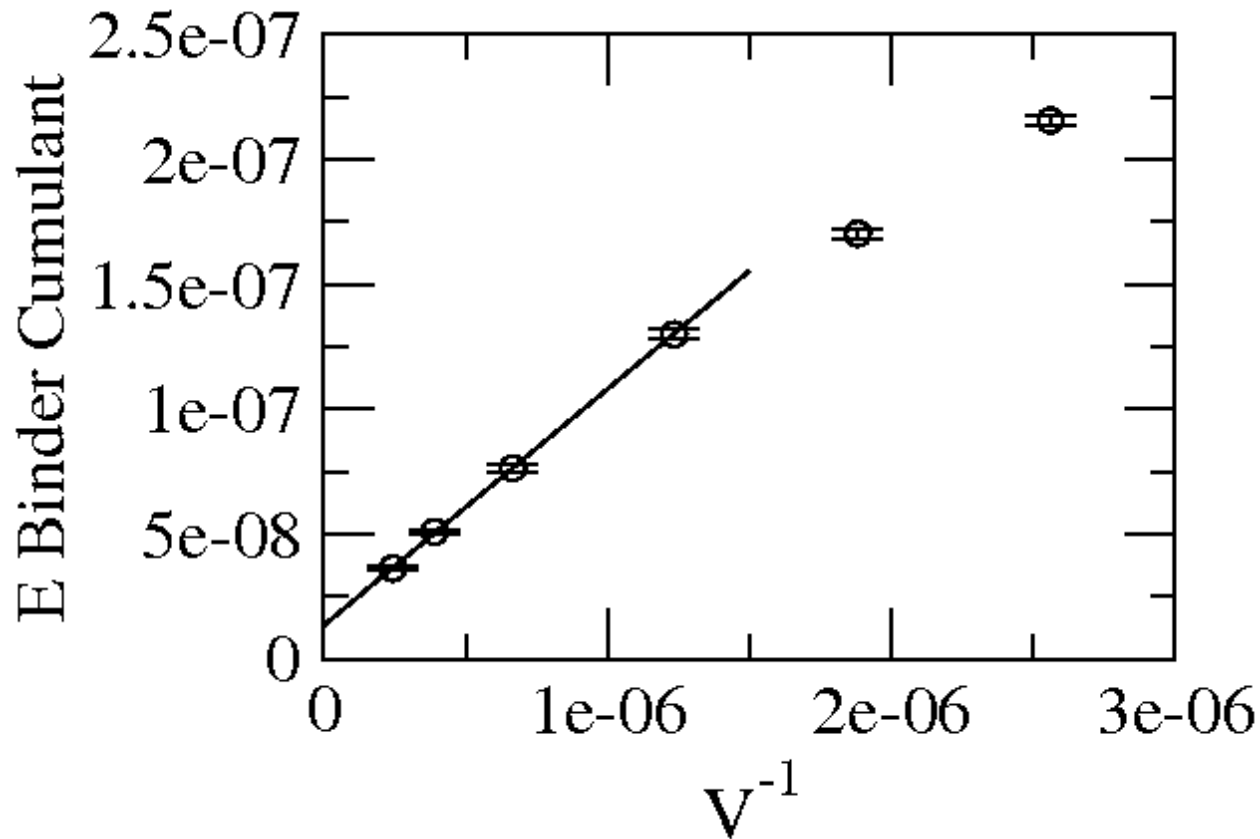


Results for “big β ” and $T = 0$ ($\beta = 2.775$)

L
30
35
40
45

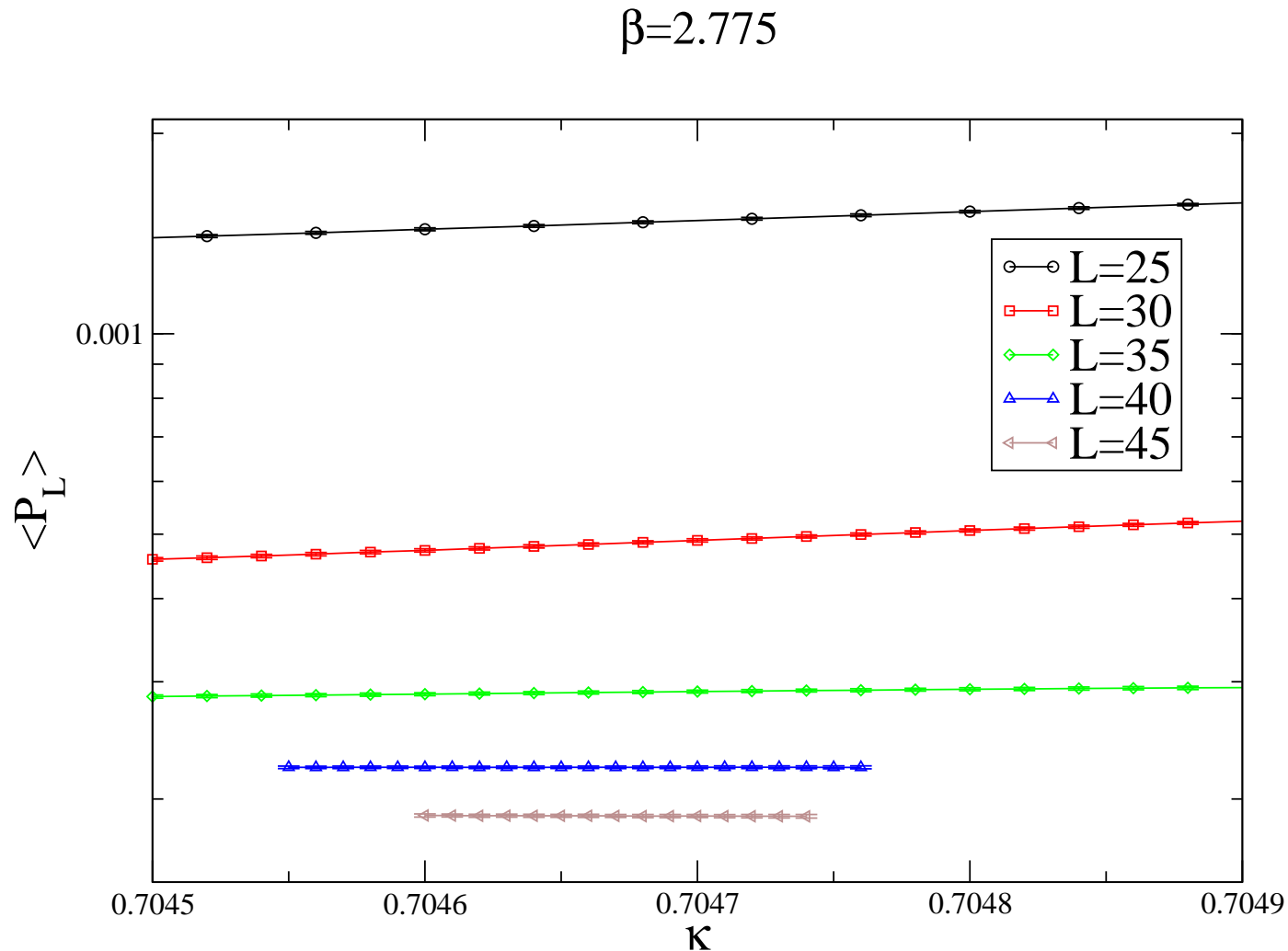


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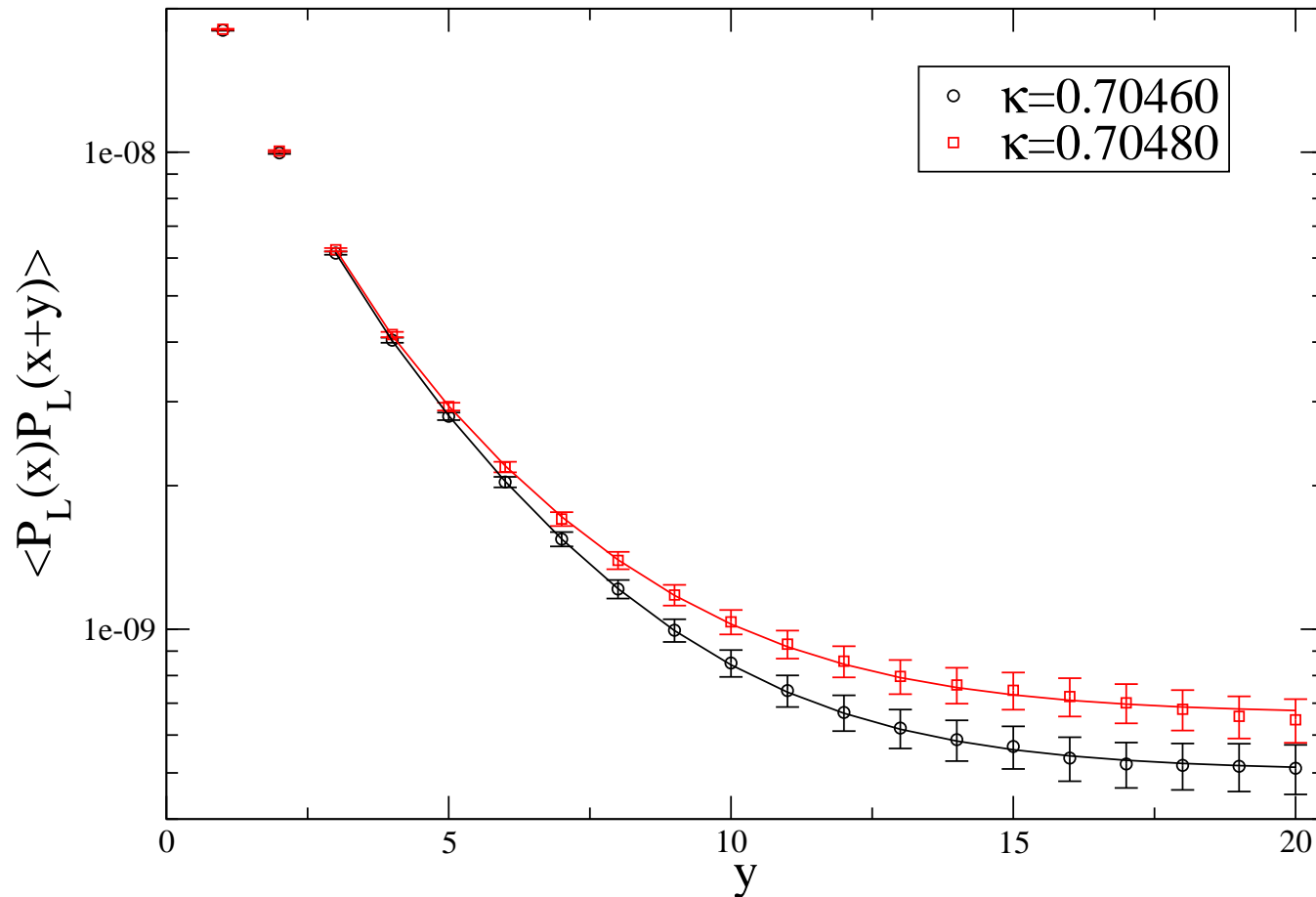
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Looking for the critical point at $T = 0$

First method

$$E_{\pm} = \lim_{\beta \rightarrow \beta_c^{\pm}} \langle E \rangle$$

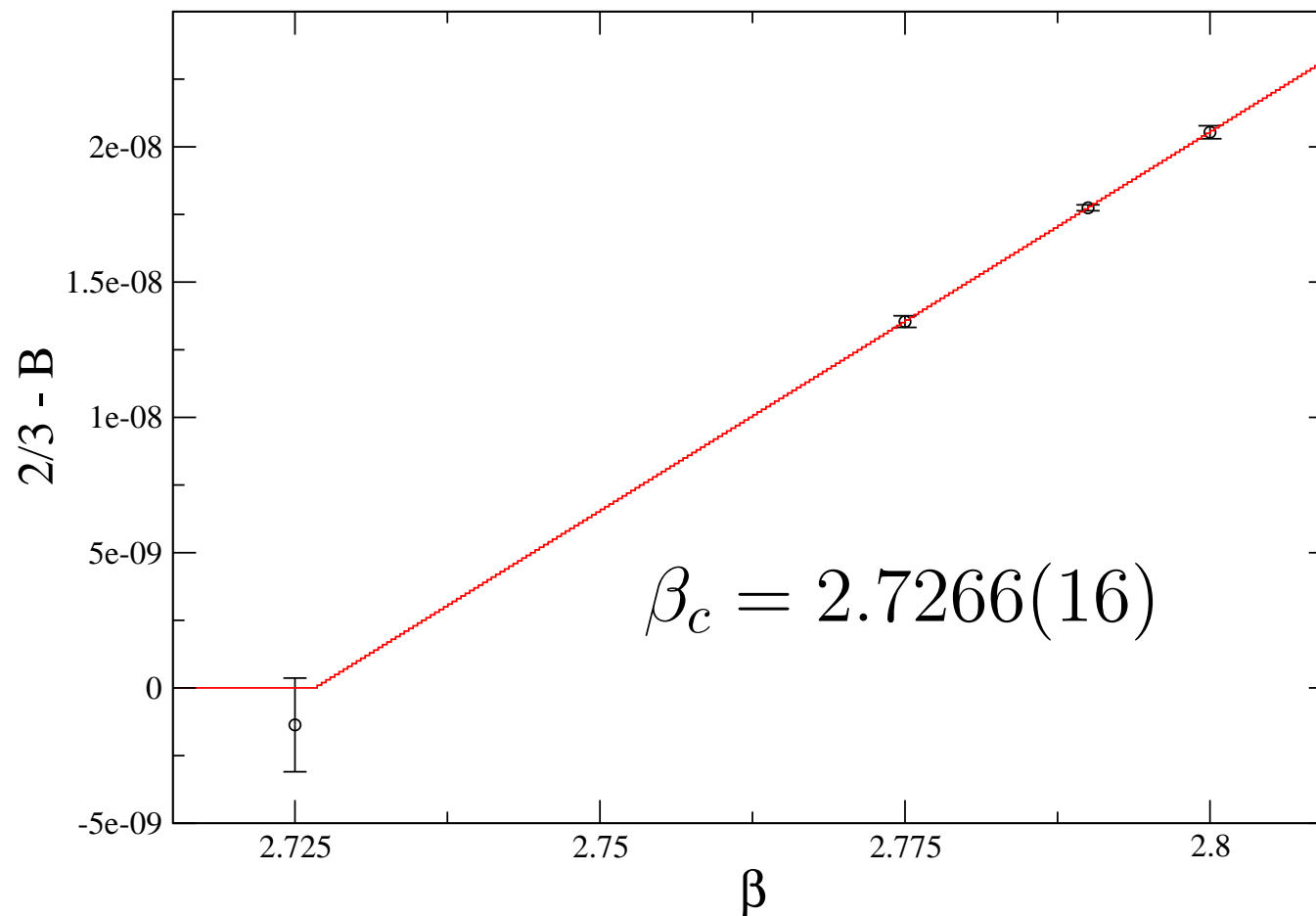
$$\Delta = E_+ - E_- \quad \epsilon = \frac{1}{2}(E_+ + E_-)$$

$$\lim_{L \rightarrow \infty} B(E) = \frac{2}{3} - \frac{1}{3} \left(\frac{\Delta}{\epsilon} \right)^2 + o(\Delta^2)$$

for a mean field critical point $\Delta \propto t^{1/2}$

$$\lim_{L \rightarrow \infty} B(E) \approx \frac{2}{3} - \gamma_1(\beta - \beta_c)$$

Looking for the critical point at $T = 0$



Looking for the critical point at $T = 0$

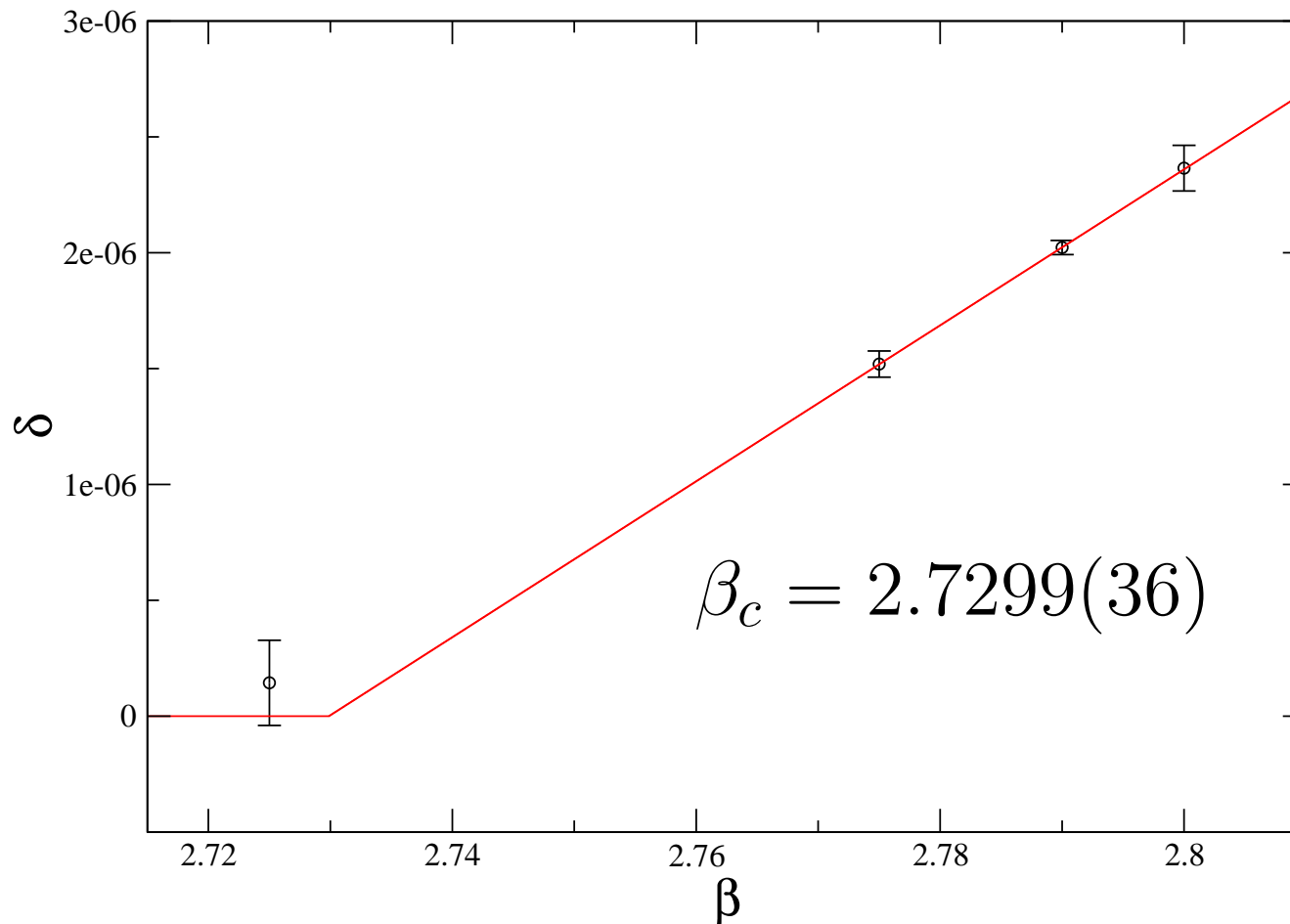
Second method

$$\chi(O) = V(\langle O^2 \rangle - \langle O \rangle^2) \propto V \Delta^2$$

for a mean field critical point $\Delta \propto t^{1/2}$

fitting $\chi = \gamma_2 + V \delta$
 $\delta \approx \gamma_3(\beta - \beta_c)$

Looking for the critical point at $T = 0$





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$$\beta_c^{(1)} = 2.7266(16)$$

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- the critical endpoint position was estimated in two independent ways
$$\beta_c^{(1)} = 2.7266(16)$$
$$\beta_c^{(2)} = 2.7299(36)$$
- the problem of the interpretation of the phase transition as a deconfinement one is raised



Conclusions

Thank you!