

Unconstrained higher-spin fields of mixed symmetry

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A. C., D. Francia, J. Mourad and A. Sagnotti, arXiv:0810.4350, arXiv:0904.4447

For a review see A. C., arXiv:0905.1472

Why Higher Spins?

- Still unclear corner of Field Theory
 - It is difficult to construct interactions for $s > 2$
 - No-go results but ...
 - ... paradigmatic examples: String Theory and interactions in *AdS* (Vasiliev construction)
- Key role in String Theory
 - No higher spins in the low-energy limit but ...
 - ... necessary for the consistency of the theory
- Different formulations of free theory could provide useful tools to treat the interaction problem

Why Mixed Symmetry?

Mixed-Symmetry Higher Spins:

- Generic fields in $D > 4$
 - Irreps of the little group $SO(D-2) \Rightarrow N \leq \frac{D-2}{2}$ labels
 - e.g. $\{3, 2\} \Rightarrow$  $\Rightarrow \varphi_{\mu_1 \mu_2 \mu_3 ; \nu_1 \nu_2}$

- Contained in the Free String spectrum

- e.g. Bosonic String:

$$|\{3, 2\}\rangle = \varphi_{\mu_1 \mu_2 \mu_3 ; \nu_1 \nu_2} \alpha_{-1}^{\mu_1} \alpha_{-1}^{\mu_2} \alpha_{-1}^{\mu_3} \alpha_{-2}^{\nu_1} \alpha_{-2}^{\nu_2} |0\rangle$$

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
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Unconstrained versus Constrained

- Different formulations for free Higher Spins:
 - local with trace constraints ←
 - non-local without constraints
 - local with auxiliary fields ←
 - unfolded Vasiliev approach
- No trace constraints in String Field Theory...
- How to eliminate the constraints?
- How to build Lagrangians?

Here focus on local formulations & massless fields



higher-spin gauge theories

Constraints

Spin s fields

- $\varphi_{\mu_1 \dots \mu_s} \longrightarrow$ unphysical polarizations, like A_μ or $h_{\mu\nu}$
 - For $s > 2$ $\delta \varphi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \Lambda_{\mu_2 \dots \mu_s)}$ is not enough!
 - For $s \geq 3$ Λ must be **traceless**
 - For $s \geq 4$ φ must be **doubly traceless**
- Algebraic constraints \Rightarrow correct counting of degrees of freedom

Mixed-symmetry fields

- $\varphi_{\mu_1^1 \dots \mu_{s_1}^1; \dots; \mu_{s_N}^N \dots \mu_{s_N}^N}$ with N “families” of symmetrized indices
 - N gauge parameters
 - Gauge for gauge transformations
 - **Constraints** on gauge parameters and fields

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Gauge invariance and constraints

- Gauge transformations

J.M.F. Labastida, 1987

- Bose: $\delta \varphi = \partial^i \Lambda_i$

- Fermi: $\delta \psi = \partial^i \epsilon_i$

- Notation

$$\varphi \rightarrow \varphi_{\mu_1 \dots \mu_{s_1}; \nu_1 \dots \nu_{s_2}; \dots}$$

$$\partial^i \Lambda_i \rightarrow \partial_{(\mu_1} \Lambda_{\mu_2 \dots \mu_{s_1})}^{(1); \nu_1 \dots \nu_{s_2}; \dots} + \partial_{(\nu_1} \Lambda_{\mu_1 \dots \mu_{s_1}; \nu_2 \dots \nu_{s_2})}^{(2); \dots} + \dots$$

- Field equations

- Bose $\mathcal{F} \equiv \square \varphi - \partial^i \partial_i \varphi + \frac{1}{2} \partial^i \partial^j T_{ij} \varphi = 0$

- Fermi $\mathcal{S} \equiv i (\not{\partial} \psi - \partial^i \psi_i) = 0$

- Gauge variation

- Bose $\delta \mathcal{F} = \frac{1}{2} \partial^i \partial^j \partial^k T_{(ij} \Lambda_{k)}$

- Fermi $\delta \mathcal{S} = -\frac{i}{2} \partial^i \partial^j \gamma_{(i} \epsilon_{j)}$

gauge invariant field
equations



(γ -)trace constraints on the
gauge parameters

Bianchi identities and constraints

- Contracted Bianchi identity in gravity $\partial^\lambda R_{\mu\lambda} - \frac{1}{2} \partial_\mu R_{\lambda}{}^\lambda = 0$

- Bianchi identities for **Bose** fields

$$\partial_i \mathcal{F} - \frac{1}{2} \partial^j T_{ij} \mathcal{F} = -\frac{1}{12} \partial^i \partial^j \partial^k T_{(ij} T_{kl)} \varphi$$

- Bianchi identities for **Fermi** fields

$$\partial_i \mathcal{S} - \frac{1}{2} \partial \not{\mathcal{S}}_i - \frac{1}{2} \partial^j T_{ij} \mathcal{S} - \frac{1}{6} \partial^j \gamma_{ij} \mathcal{S} = \frac{i}{6} \partial^j \partial^k T_{(ij} \not{\psi}_{k)}$$

- Gauge variation of the linearized Einstein-Hilbert Lagrangian

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \quad \Rightarrow \quad \delta \mathcal{L} = -\frac{1}{2} \xi^\mu \left(\partial^\lambda R_{\mu\lambda} - \frac{1}{2} \partial_\mu R_{\lambda}{}^\lambda \right)$$

Lagrangians \Leftrightarrow (γ -)trace constraints on the fields

Permutation group & Lagrangians

- Constraints on the kinetic tensors
 - Bose $T_{(ij} T_{kl)} \varphi = 0 \Rightarrow T_{(ij} T_{kl)} \mathcal{F} = 0$
 - Fermi $T_{(ij} \gamma_k) \psi = 0 \Rightarrow T_{(ij} \gamma_k) \mathcal{S} = 0$
- Non-vanishing combinations of multiple (γ -)traces of the kinetic tensors
 \Rightarrow they can and indeed do enter the Lagrangians

- Lagrangians for **Bose** fields

J.M.F. Labastida, 1989

$$\mathcal{L} = \frac{1}{2} \langle \varphi, \sum_{p=0}^N \frac{(-1)^p}{p!(p+1)!} \eta^p \mathcal{F}^{[p]} \rangle$$

- Lagrangians for **Fermi** fields

A. C., D. Francia, J. Mourad, A. Sagnotti, 2009

$$\mathcal{L} = \frac{1}{2} \langle \bar{\psi}, \sum_{p,q=0}^N \frac{(-1)^{p+\frac{q(q+1)}{2}}}{p!q!(p+q+1)!} \eta^p \gamma^q (\gamma^{[q]} \mathcal{S}^{[p]}) \rangle$$

- Structure \Rightarrow permutation group acting on “family” indices
Coefficients \Rightarrow **Bianchi identities and their (γ -)traces**

Gauge invariance & auxiliary fields

D. Francia, A. Sagnotti, J. Mourad, 2002-2007

A. C., D. Francia, J. Mourad, A. Sagnotti, 2008-2009

• Field equations

- Bose $\delta \mathcal{F} = \frac{1}{2} \partial^i \partial^j \partial^k T_{(ij} \Lambda_{k)}$

- Fermi $\delta \mathcal{S} = -\frac{i}{2} \partial^i \partial^j \gamma_{(i} \epsilon_{j)}$

• Lagrangians

- Bose $\delta \mathcal{L}_{\text{Bose}} = -\frac{1}{96} \langle \partial_{(i} \partial_j \partial_k \Lambda_{l)} + \dots, T_{(ij} T_{kl)} \varphi \rangle$

- Fermi $\delta \mathcal{L}_{\text{Fermi}} = -\frac{i}{36} \langle \partial_{(i} \partial_j \epsilon_{k)} + \dots, T_{(ij} \gamma_k) \psi \rangle$

• Compensators

- Bose $\delta \Phi_i = \Lambda_i \longrightarrow \mathcal{A} = \mathcal{F} - \frac{1}{6} \partial^i \partial^j \partial^k T_{(ij} \Phi_{k)}$

- Fermi $\delta \Psi_i = \epsilon_i \longrightarrow \mathcal{W} = \mathcal{S} + \frac{i}{2} \partial^i \partial^j \gamma_{(i} \Psi_{j)}$

• Lagrange multipliers

$$\tilde{\mathcal{L}}_{\text{Bose}} = \mathcal{L}_{\text{Bose}} + \frac{1}{24} \langle \beta_{ijkl}, T_{(ij} T_{kl)} \varphi \rangle \quad \delta \beta_{ijkl} = \frac{1}{4} \partial_{(i} \partial_j \partial_k \Lambda_{l)} + \dots$$

$$\tilde{\mathcal{L}}_{\text{Fermi}} = \mathcal{L}_{\text{Fermi}} + \frac{i}{12} \langle \bar{\lambda}_{ijk}, T_{(ij} \gamma_k) \psi \rangle \quad \delta \lambda_{ijk} = \frac{1}{3} \partial_{(i} \partial_j \epsilon_{k)} + \dots$$

Weyl-like symmetries

- Do the Lagrangian equations of motion reduce to $\mathcal{F} = 0$ or to $\mathcal{S} = 0$?
 - YES, for $D > 2N + 1$ but ...
 - ... remember gravity in $D = 2$
 - Off-shell $R_{\mu\nu} \neq 0$ but $R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R \equiv 0$
 - Weyl symmetry: $\delta h_{\mu\nu} = \eta_{\mu\nu} \Omega$
- In particular space-time dimensions $D \leq 2N + 1$ some Lagrangians display new Weyl-like symmetries $\delta \varphi = \eta^{ij} \Omega_{ij}$

D	$N \leq 2$ bosons
2	<i>gravity</i>
2	{3, 1}
3	{2, 1}
3	{2, 2}
4	{s, 2}

D	$N \leq 2$ fermions
2	<i>gravitino</i>
2	{2, 2}
3	{1, 1}
4	{s, 1}

Summary & Outlook

- Main ideas and tools
 - Gauge invariance
 - Bianchi identities
 - Permutation group acting on “family” indices
- New results
 - Constrained Lagrangians for Fermi fields
 - Unconstrained Lagrangians for Bose and Fermi fields
 - Weyl-like symmetries
- Perspectives
 - Extension to $(A)dS$, Supersymmetry
 - Links with String Theory
 - Interactions. . .