

Finite size phase transitions in QCD with adjoint fermions

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Summary of the talk

- 1 Motivation and background
 - Introduction - Large N_C theories
 - QCD with Adjoint fermions on the lattice
- 2 Results
 - Phase space diagram - Chiral limit
 - Chiral properties
 - Summary and outlook

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Introduction

Paper: G.Cossu, M.D'Elia arXiv:0904.1353 [hep-lat]

Large N_c theories

- Useful tool for the study of non perturbative QCD
- QCD with fermions in the antisymmetric (AS) representation is a different $N_c \rightarrow \infty$ limit of ordinary QCD (equal at $N_c = 3$).
- Orientifold equivalence \Rightarrow QCD(AS) = QCD(Adj) in this limit (Shifman, Armoni, Veneziano, Patella)
- Kovtun, Unsal, Yaffe claims that QCD(Adj) stays confined in a $\mathbb{R}^3 \times S_1$ space if we shrink the compact dimension. Easier calculations extrapolated back to ordinary QCD by the various equivalences.

QCD(Adj) at small volume - 1

Kovtun, Unsal, Yaffe JHEP, 0706 (2007) 019

Analysis of 1-loop effective potential for the Polyakov Line along the compact dimension (L_C), N_f Majorana fermions.

Gives all the relevant information at small L_C .

Polyakov Loop Potential

$$V_{\text{eff}}(\Omega) = \frac{1 \mp N_f}{24\pi^2 L_C^4} \sum_{i,j=1}^{N_C} [\phi_i - \phi_j]^2 ([\phi_i - \phi_j] - 2\pi)^2 \quad (1)$$

+/- for periodic/antiperiodic boundary conditions (PBC/ABC)
 e^{ϕ_i} eigenvalues of the Polyakov Loop.

- ABC or $N_f = 0$ minimized if eigvals coincide
- PBC: repulsive potential for eigvals, disordered.

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QCD(Adj) at small volume - 2

- Results confirmed by 1-loop lattice calculations
- Same conclusions in a $SU(N_C)$ + trace deformation: i.e. small and large volumes are analytically connected.
- The essential requirement is that **no phase transition occurs**: we will address this point by numerical lattice simulations of QCD with 2 Dirac adjoint fermions and $N_C = 3$.
- We will discuss the chiral and continuum limit.
- We conclude that the claim by Kovtun, Unsal and Yaffe is highly non trivial and should be considered with care.

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Lattice discretization of QCD(Adj)

Adjoint Fermions

- 8 color d.o.f. $Q = Q^a \lambda_a$ (λ_a Gell-Mann's matrices)
- $U_{(8) i, \mu}^{ab} = \frac{1}{2} \text{Tr} \left(\lambda^a U_{(3) i, \mu} \lambda^b U_{(3) i, \mu}^\dagger \right)$ (8-dim rep. of links)
- $S = S_G [U_{(3)}] + \sum_{i,j,a,b} \bar{Q}_i^a M [U_{(8)}]_{i,j}^{a,b} Q_j^b$ (Action)
- Staggered fermions

Partition Function

- $Z = \int \mathcal{D}U \mathcal{D}\Phi_e e^{-S_G} \exp \left(-\Phi_e^t (M^t M)_{ee}^{-1} \Phi_e \right)$
- Integral sampled by standard Hybrid Monte Carlo Φ algorithm.

Center symmetry and Polyakov Loop

Center symmetry

YM theories are invariant if we multiply all links in a temporal slice by a center element. Fermionic terms (in the fundamental representation) break this symmetry

Polyakov Loop

$$L_3 = \text{Tr} \prod_{t=0}^{t=N_t-1} U_3(\vec{x}, t) \quad (2)$$

- The action of QCD(Adj) is invariant for center symmetry.
- The Fundamental Polyakov Loop is not and therefore is an order parameter for its spontaneous breaking.

Simulations parameters - 1

- ABC : thermodynamical system.
- PBC : no thermodynamical interpretation although we will use the same terminology.
- We simulated the variation of length of the compactified dimension by varying β (gauge coupling) and keeping the number of sites constant.
- By asymptotic freedom lattice spacing $a \rightarrow 0$ as $\beta \rightarrow \infty$.
- At infinite mass (pure YM) we have two distinct phases separated by a first order transition.
- At the chiral point we have two confined phases at the opposite limits of compact dimension ($L_c \rightarrow 0, \infty$).
- A highly non trivial phase structure must be found at intermediate masses.

Simulations parameters - 2

- Several simulations with different masses toward the chiral limit are essential ($am = 0.50, 0.10, 0.05, 0.02, 0.01$).
- Lattice dimensions $16^3 \times L_C$ with $L_C = 4$ for most of the simulations.

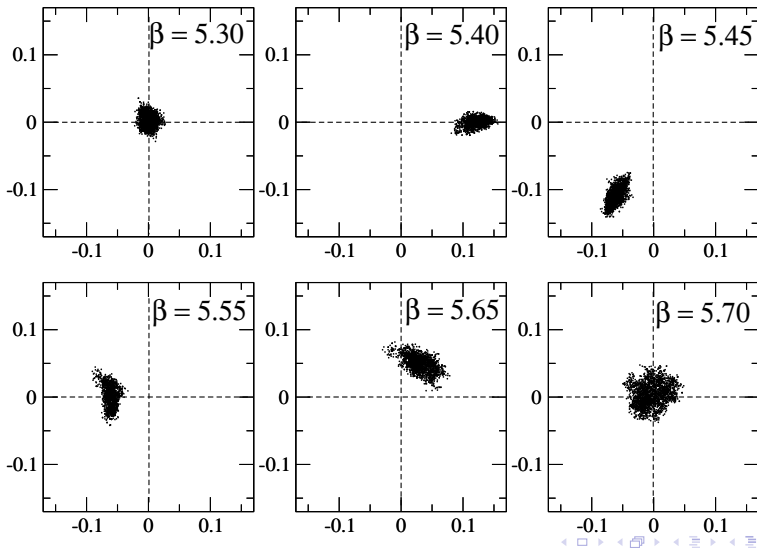
Technicalities

- Trajectory length: 0.5
- Integration step δt from 0.025 to 0.005 depending on the mass to keep acceptance rate around 80%
- MC data from 2k to 20k depending on parameters
- Performed on the APEmille installation in Pisa and the apeNEXT facility in Rome.

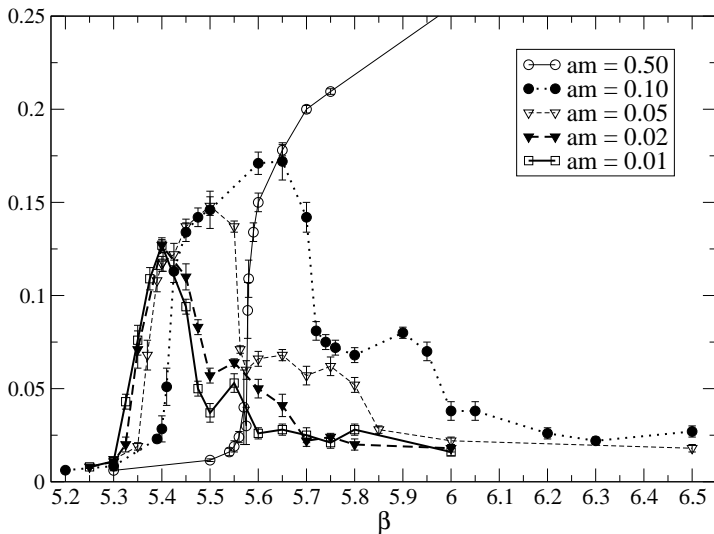
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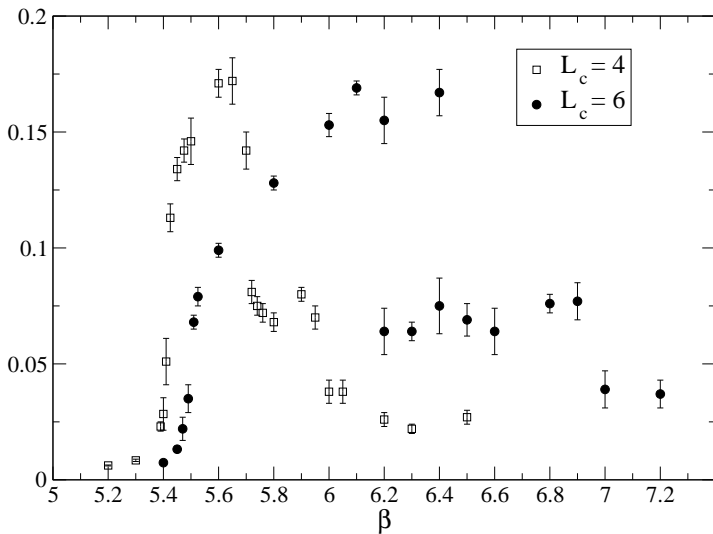
Phase transitions $am=0.02$



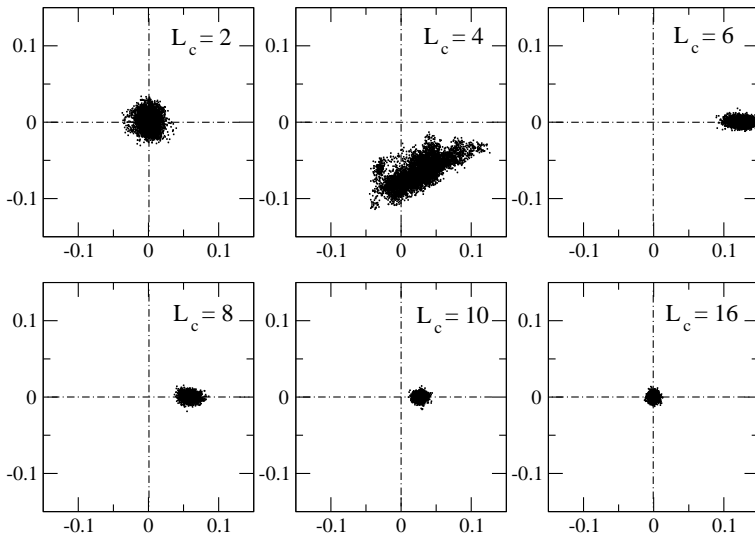
Phase transitions - Cumulative



Phase transitions - Continuum Limit $16^3 \times 6$

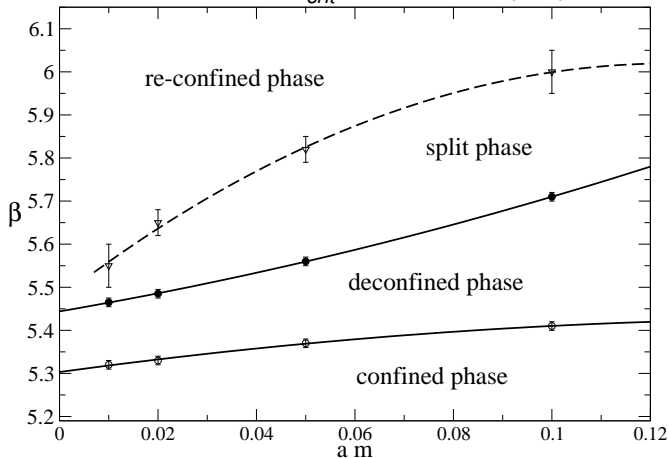


Phase transitions - Fixed lattice spacing



Chiral limit extrapolations

Extrapolation : $\beta_{crit} = \beta_{crit}^{\chi} + c_1 am + c_2 (am)^2$

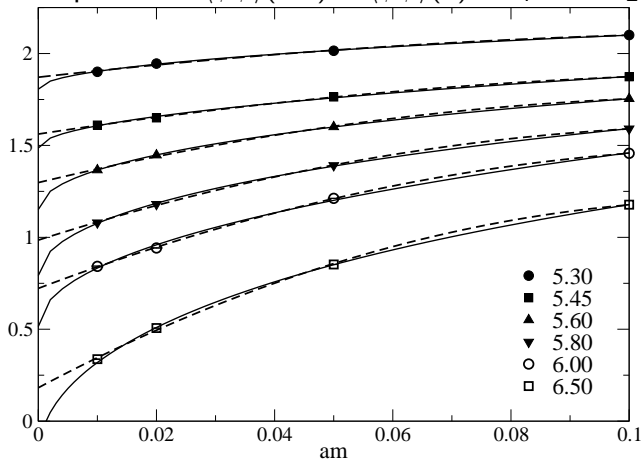


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Chiral condensate extrapolations

$$\text{Extrapolation : } \langle \bar{\psi}\psi \rangle(am) = \langle \bar{\psi}\psi \rangle(0) + a_1 am + a_2(am)^2$$



Relation with center symmetry - 1

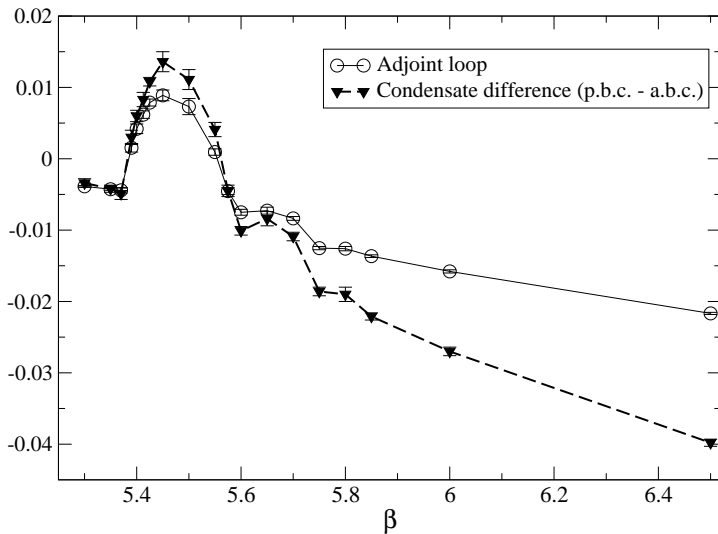
- Bilgici, Bruckmann, Gatringer, Hagen proposed a new operator to detect confinement that is related to the chiral properties.

$$\Sigma_1 = -\frac{1}{2\pi} \int_0^{2\pi} d\varphi e^{-i\varphi} \langle \bar{\psi}\psi \rangle_m^{(\varphi)} \quad (3)$$

where $\langle \bar{\psi}\psi \rangle_m$ is the chiral condensate measured with an assigned phase for the fermionic boundary conditions in the compactified dimension. In pure YM this reduces to ordinary Polyakov Loop.

- We measured an approximation : $(\langle \bar{\psi}\psi \rangle_m^{(0)} - \langle \bar{\psi}\psi \rangle_m^{(\pi)})$

Relation with center symmetry - 2



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Conclusions

- We have shown that the claim by Kovtun, Unsal and Yaffe must be taken with care.
- At least two phase transitions seem to occur at small masses for PBC QCD(Adj) if we shrink the volume toward small compactification radii.
- A new phase appear at intermediate masses (it is not clear if it survives the chiral limit)
- We verified that center symmetry is reflected in the dependence of the chiral condensate on BC.
- Chiral limit must be further investigated and different discretization methods would be useful to detect systematic errors.