

ATOMIC JOSEPHSON JUNCTION WITH TWO BOSONIC SPECIES

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Atomic Josephson junction (AJJ) with two bosonic species

The prediction of Bose-Einstein condensation (1924) and its realization in the laboratory (1995) paved the way for many important theoretical and experimental developments. One of these is the atomic counterpart of the Josephson effect which occurs in superconductor-oxide-superconductor (S-O-S) junctions (1962). In a S-O-S junction, the onset of a Cooper pairs electrical current occurs through the oxide layer.

$$\psi^\alpha(t) = \sqrt{\rho^\alpha(t)} \exp(i\theta^\alpha(t)); \alpha = L, R$$

$$\frac{d}{dt} \rho^{L,R}(t) = \pm \frac{2K}{\hbar} \sqrt{\rho^L \rho^R} \sin(\theta^L - \theta^R)$$

$$\frac{d}{dt} \theta^L(t) = \frac{K}{\hbar} \sqrt{\rho^R / \rho^L} \cos(\theta^L - \theta^R)$$

$$\frac{d}{dt} \theta^R(t) = \frac{K}{\hbar} \sqrt{\rho^L / \rho^R} \cos(\theta^L - \theta^R)$$

Atomic Josephson junction (AJJ) with two bosonic species

Under certain conditions, a coherent transfer of matter consisting of a single Bose-Einstein condensate (BEC) atoms flows across the junction. Single BEC confined by a double well potential (Leggett *et al* (1991); Zapata *et al* (1998); Smerzi *et al* (1997)) . Gross-Pitaevkii equation (GPE)

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + [V_{trap}(\mathbf{r}) + g |\Psi|^2] \Psi, \iiint dx dy dz |\Psi(x, y, z)|^2 = N, g = 4\pi\hbar^2 a / m$$

Two mode approximation and ordinary differential equations (ODEs)

$$\Psi = \psi^L(t) \phi^L(\mathbf{r}) + \psi^R(t) \phi^R(\mathbf{r})$$

$$\psi^\alpha(t) = \sqrt{N^\alpha(t)} \exp(i\theta^\alpha(t)), N = N^L + N^R$$

(AJJ) with two bosonic species

Two mode approximation and ordinary differential equations (ODEs).

$$\frac{d z}{d t} = - \frac{2}{\hbar} K \sin \theta \sqrt{1 - z^2}$$

$$\frac{d \theta}{d t} = \frac{U N z}{\hbar} + \frac{2 K z}{\hbar \sqrt{1 - z^2}} \cos \theta$$

$$z = (N^L - N^R) / N, \theta = \theta^R - \theta^L$$

When the atom-atom interaction is strong enough, the fractional imbalance z oscillates in the time around a nonzero time averaged value: macroscopic quantum self-trapping (MQST).

AJJ with two bosonic species

In presence of two BECs with intra- and inter-species interactions

Xu et al (2008);

Satija et al (2009).

Extension of the ODEs to the two bosonic component case. A comparison between the predictions of the GPE and the ones of the ODEs equation lacks.

AJJ with two bosonic species

We consider two BECs, each of them made of repulsively interacting. We have two different bosonic components ($i=1,2$).

We suppose that two BECs -attractively interacting with each other- are confined by the trapping potential (strong harmonic confinement in the x - y plane + double-well potential in the z -direction)

$$\mathbf{V}_{\text{trap}}(\mathbf{r}) = \frac{m_i \omega^2}{2} (x^2 + y^2) + V_{DW}(z)$$

AJJ with two bosonic species

For symmetric configurations in the z -direction, we model the double-well potential as

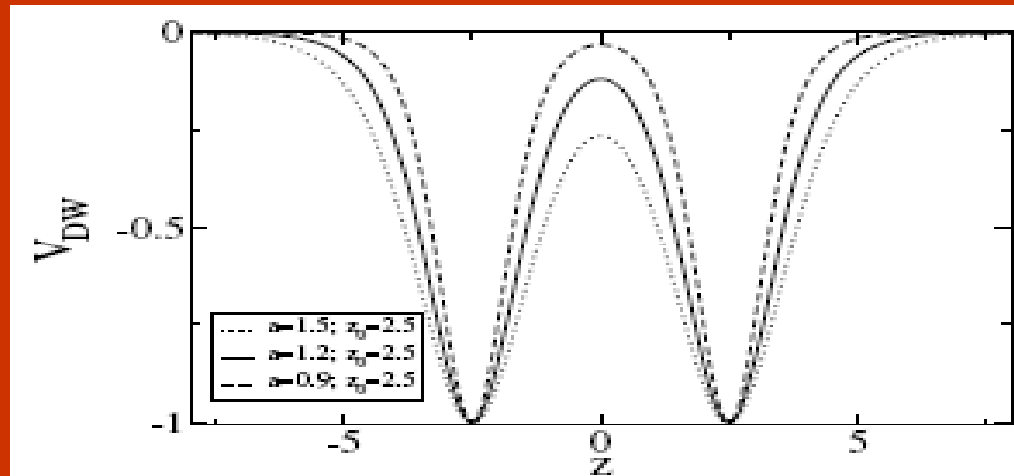
$$V_{DW}(z) = V_L(z) + V_R(z)$$

$$V_{L,R}(z) = -V_0 \left[\text{Sech}^2 \left(\frac{z \pm z_0}{a} \right) \right]$$

$$V_0 = \hbar \omega_i \left[1 + \text{Sech}^2 \left(\frac{2z_0}{a} \right) \right]^{-1}$$

The left part (L) and right part (R) are separated by a potential barrier, the height of which can be changed by varying a . The L and R parts are centred around the points $-z_0$ and z_0 .

AJJ with two bosonic species



The wavefunction of the ground state, for instance, for the only left well potential is

$$\phi_i^{(L,PT)}(z) = A \left[1 - \text{Tanh}^2 \left(\frac{z + z_0}{a} \right) \right]^{B_i/2}$$

$$B_i = -\frac{1}{2} + \sqrt{\frac{2m_i V_0 a^2}{\hbar^2} + \frac{1}{4}}$$

A ensures the normalization of the wavefunction.

AJJ with two bosonic species

In the mean-field approximation, the spatio-temporal evolution of the two BECs is controlled by the two coupled Gross-Pitaevskii equations (GPEs)

$$i\hbar \frac{\partial \Psi_i}{\partial t} = -\frac{\hbar^2}{2m_i} \Delta \Psi_i + \left[V_{trap}(\mathbf{r}) + g_i |\Psi_i|^2 + g_{ij} |\Psi_j|^2 \right] \Psi_i$$

$$\iiint dx dy dz |\Psi_i(x, y, z)|^2 = N_i$$

$$g_i = 4\pi\hbar^2 a_i / m_i$$

$$g_{ij} = 2\pi\hbar^2 a_{ij} / m_r$$

$$m_r = m_i m_j / (m_i + m_j)$$

AJJ with two bosonic species

We are interested in studying the dynamical oscillations of the populations of each condensate between the L and R wells.

The idea is writing the Lagrangian associated to the previous GPEs

$$L = \int d^3 \mathbf{r} \sum_{i=1,2} \Psi_i^* \left(i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m_i} \Delta \right) \Psi_i - V_{trap}(\mathbf{r}) |\Psi_i|^2 - \frac{g_i}{2} |\Psi_i|^4 - g_{ij} |\Psi_i|^2 |\Psi_j|^2$$

and adopt the ansatz (Salasnich *et al*, 2006)

$$\Psi_i(x, y, z, t) = \frac{1}{\sqrt{\pi} a_{\perp,i}} \exp \left[-\frac{x^2 + y^2}{2 a_{\perp,i}^2} \right] f_i(z, t)$$

$$a_{\perp,i} = \sqrt{\frac{\hbar}{m_i \omega_i}}$$

$$\int_{-\infty}^{+\infty} dz |f_i(z, t)|^2 = N_i$$

AJJ with two bosonic species

Following the path displayed in the previous slide, we exploit the strong confinement in the x - y plane in to get the effective 1D equations describing the dynamics in the z -direction. By performing the integration in the x - y plane, we obtain the effective Lagrangian 1D for the fields f_i . By varying such a Lagrangian with respect to the complex conjugate of f_i , we obtain the 1D GPEs for the field f_i

$$i\hbar \frac{\partial f_i}{\partial t} = -\frac{\hbar^2}{2m_i} \frac{d^2 f_i}{dz^2} + \left[V_{DW}(z) + \tilde{g}_i |f_i|^2 + \tilde{g}_{ij} |f_j|^2 \right] f_i$$

$$\tilde{g}_i = \frac{g_i}{2\pi a_{\perp,i}^2}$$

$$\tilde{g}_{ij} = \frac{g_{ij}}{\pi (a_{\perp,i}^2 + a_{\perp,j}^2)}$$

AJJ with two bosonic species

It is possible to study the AJJ dynamics described the previous 1D GPEs coupled equations by using the two mode approximation.

We assume for each f_i the following time-dependent wavefunction decomposition

$$f_i(z, t) = \psi_i^L(t) \phi_i^L(z) + \psi_i^R(t) \phi_i^R(z)$$

$$\psi_i^\alpha(t) = \sqrt{N_i^\alpha(t)} \exp(i\theta_i^\alpha(t))$$

$$N_i^L + N_i^R = |\psi_i^L(t)|^2 + |\psi_i^R(t)|^2 = N_i$$

$$\int_{-\infty}^{+\infty} dz |\phi_i^\alpha(z)|^2 = 1, \quad \int_{-\infty}^{+\infty} dz \phi_i^L(z) \phi_i^R(z) = 0$$

AJJ with two bosonic species

Neglecting terms of ϕ 's overlaps between different wells, we get

$$\begin{aligned} \bar{L} = & \sum_{i=1,2} -\hbar \dot{\theta}_i^L - \hbar \dot{\theta}_i^R - E_i^L N_i^L - E_i^R N_i^R + \\ & 2 K_i \sqrt{N_i^L N_i^R} \cos(\theta_i^L - \theta_i^R) \\ & - \left(\frac{U_i^L}{2} (N_i^L)^2 + \frac{U_i^R}{2} (N_i^R)^2 \right) - U_{12}^L N_1^L N_2^L - U_{12}^R N_1^R N_2^R \\ E_i^\alpha = & \int dz \frac{\hbar^2}{2 m_i} \left(\frac{d \phi_i^\alpha}{dz} \right)^2 + \left(V_{DW} + \frac{\hbar^2}{2 m_i a_{\perp,i}^2} + \frac{m_i \omega_i^2 a_{\perp,i}^2}{2} \right) (\phi_i^\alpha)^2 \\ U_i^\alpha = & \tilde{g}_i \int dz (\phi_i^\alpha)^4, U_{12}^\alpha = \tilde{g}_{ij} \int dz (\phi_1^\alpha)^2 (\phi_2^\alpha)^2 \\ K_i = & \int dz \left(- \frac{\hbar^2}{2 m_i} \frac{d \phi_i^L}{dz} \frac{d \phi_i^R}{dz} + V_{DW} \phi_i^L \phi_i^R \right) \end{aligned}$$

In the following, we shall consider two isotopes of the same alkali-atom so that the difference mass can be neglected (Papp *et al*, 2006)

AJJ with two bosonic species

The functions ϕ 's (L,R) may be calculated from the eigenvalue problem corresponding to the 1D GPE with g 's=0.

By means a perturbative approach (V_L perturbed by the presence of V_R) we get

$$\phi_i^L = \frac{(MS + MA)\phi_i^{(L,PT)} + (MS - MA)\phi_i^{(R,PT)}}{\sqrt{2}}$$
$$\phi_i^R = \frac{(MS - MA)\phi_i^{(L,PT)} + (MS + MA)\phi_i^{(R,PT)}}{\sqrt{2}}$$

With this ϕ 's (L,R), we evaluate the E 's U 's and the K 's (proportional to the Rabi frequency)

With $a=1.2$ and $z0=2.5$, we get $K1=K2=K=0.0148$.

AJJ with two bosonic species

The equations of motion descending from the 1D effective Lagrangian written in terms of $N_i^\alpha, \theta_i^\alpha$ lead the following system of ordinary differential equations (ODEs, coupled pendula) for each bosonic component

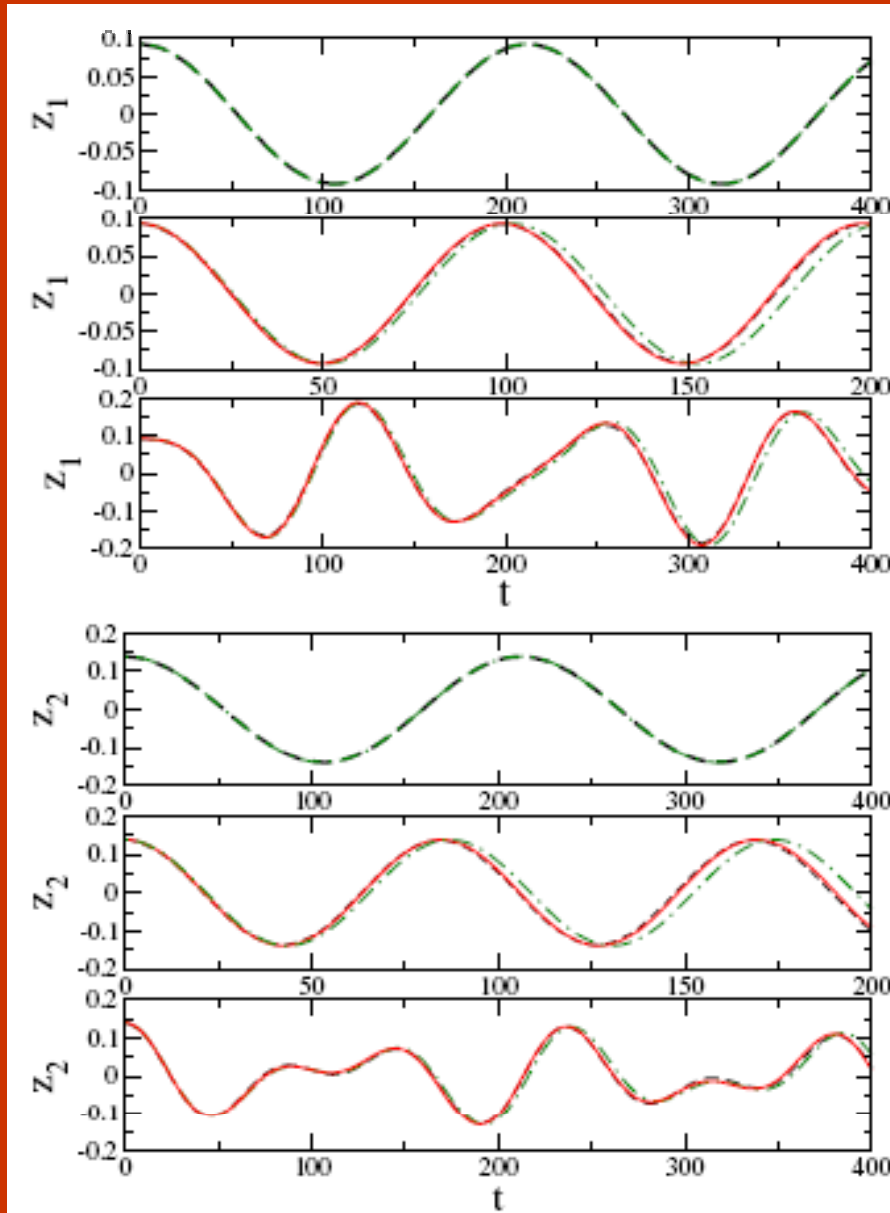
$$\frac{dz_i}{dt} = -\frac{2}{\hbar} K_i \sin \theta_i \sqrt{1 - z_i^2}$$

$$\frac{d\theta_i}{dt} = \frac{U_i N_i z_i}{\hbar} + \frac{2 K_i z_i}{\hbar \sqrt{1 - z_i^2}} \cos \theta_i + \frac{U_{12} N_j z_j}{\hbar}$$

$$z_i = (N_i^L - N_i^R) / N_i, \theta_i = \theta_i^R - \theta_i^L$$

We assumed the two wells be symmetric so that E 's, U 's, K 's. do not depend on the index α

AJJ with two bosonic species



*First panel of each z : no interaction at all

*Second panel: only intra-species interaction finite

*Third panel: intra and inter-species interaction finite

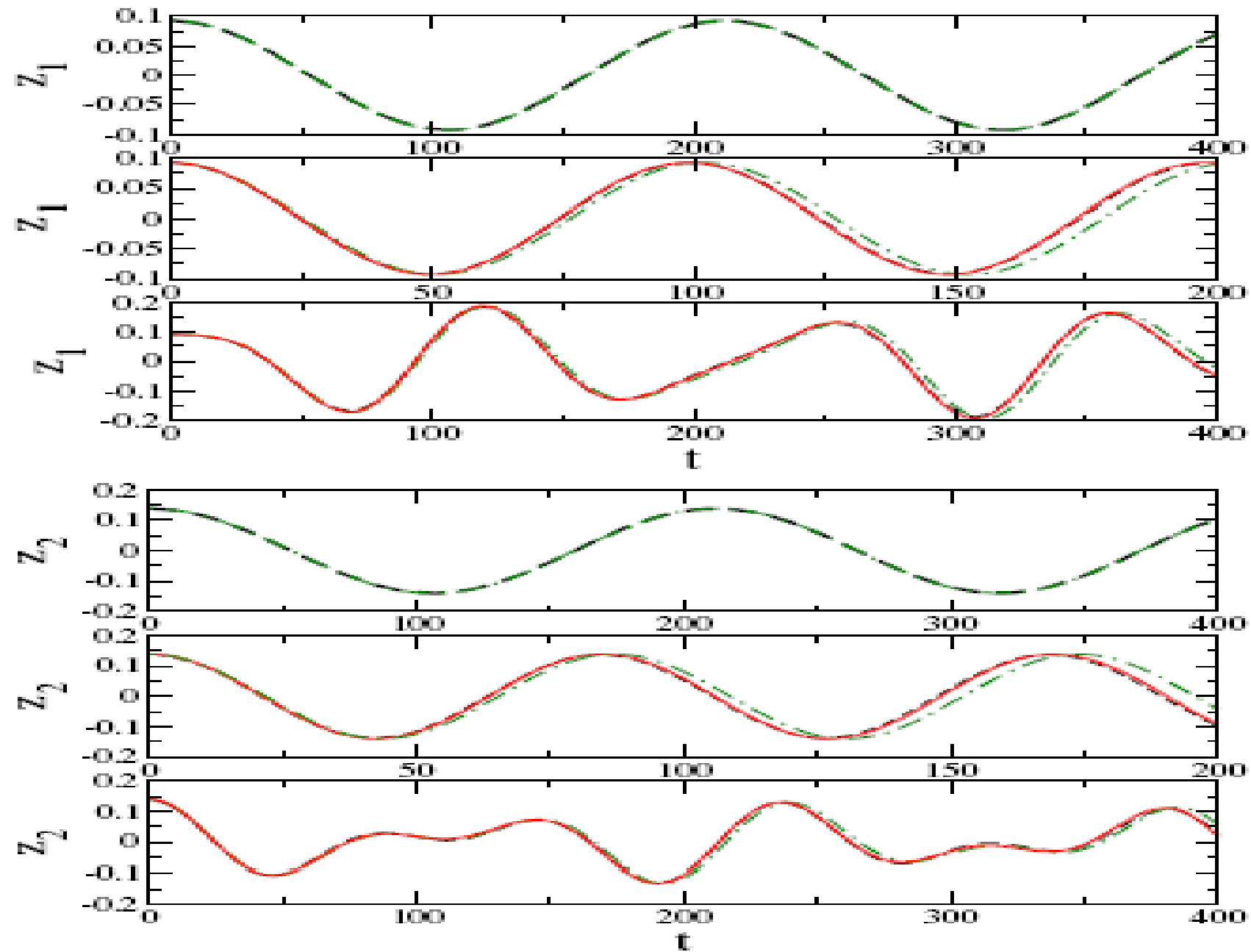
*Dashed line: integration of the two GPEs

*Dot-dashed line: integration of the four ODEs with $K=0.0148$

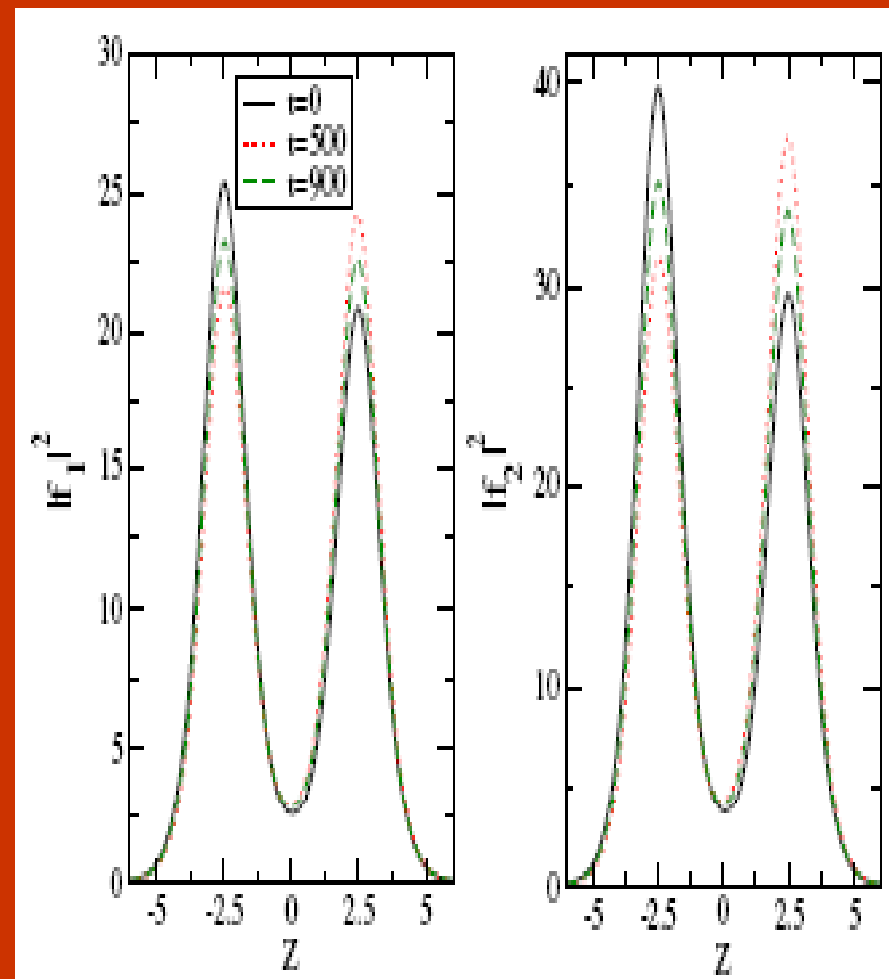
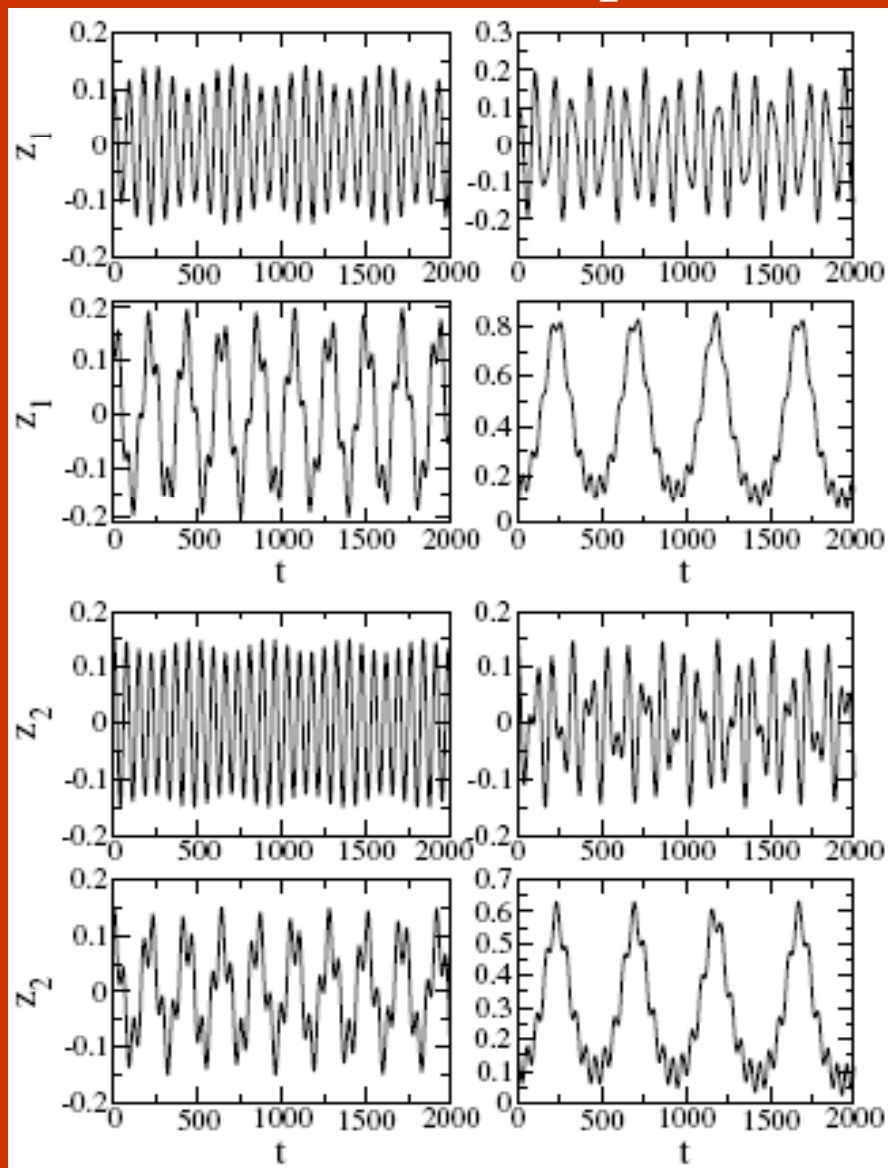
*Continuous line: integration of the four ODEs with a best fit K , say K_{bf} .

For the second panels $K_{bf}=0.0155$

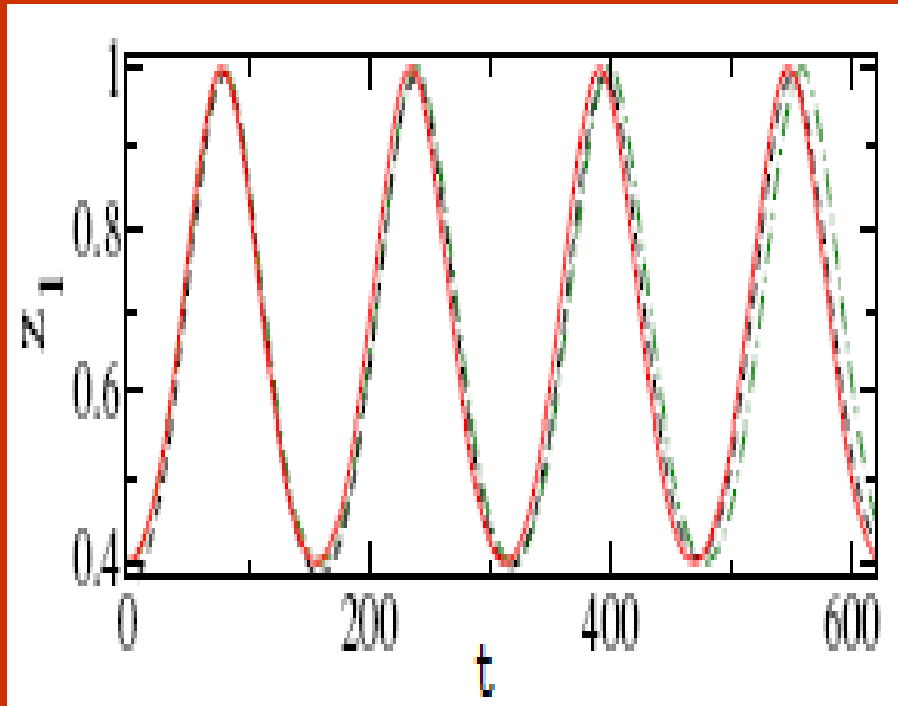
For the third panels $K_{bf}=0.0151$



AJJ with two bosonic species
(the intra-species interaction is fixed, while the inter-species one changes)



AJJ with two bosonic species



*Dashed line: integration of the two GPEs

*Dot-dashed line: integration of the four ODEs with $K=0.0148$

*Continuous line: integration of the four ODEs with a best fit K , say K_{bf} . $K_{bf}=0.0151$

Conclusions.

We have analyzed the atomic Josephson effect with a binary mixture of BECs.

We have shown that in correspondence with precise values of the inter-species interaction amplitude, the relative populations oscillate around a nonzero time averaged value, i.e. the MQST.

We have compared the predictions of GPEs with those of the ODEs. Under certain conditions, the two predictions agree with each other: the ODEs describe, consistently, the atomic Josephson junction physics.