

*Equilibrium sequences of hybrid  
stars with LOFF matter core*

**Nicola Daniele IPPOLITO**

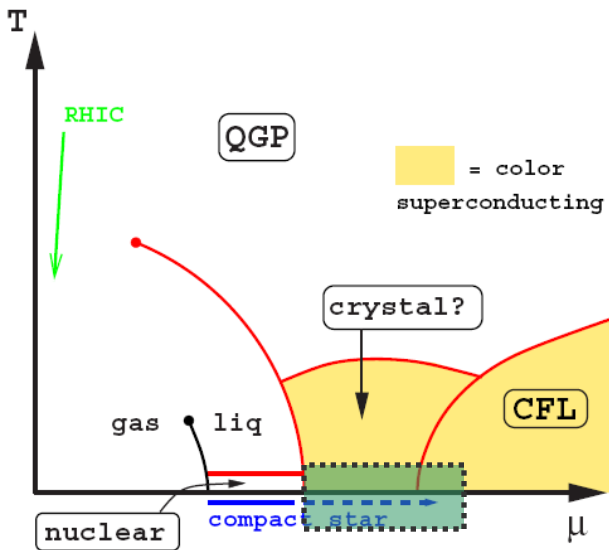
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# Summary

- Introduction to the crystalline color superconducting phases (LOFF phases) with three flavors;
- Equation of State of crystalline phases with three flavors;
- Equilibrium sequences of hybrid compact stars with LOFF core;
- Outlook.

# The LOFF color superconducting phase with three flavors

R.Casalbuoni, R.Gatto, **N.D.Ippolito**, G.Nardulli, M.Ruggieri  
 Phys. Lett. B **627**, 89 (2005)



For high densities and very low temperatures, it is believed that quark matter manifests a coupling among quarks similar to the Cooper-pairing mechanism of ordinary electromagnetic superconducting materials. Therefore this phenomenon is named **Color SuperConductivity (CSC)**.

Ansatz for the condensate in the three flavor LOFF phase:

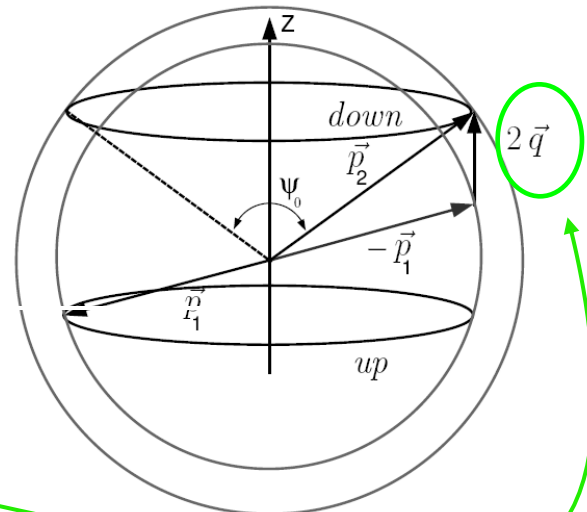
$$\langle \psi_{i\alpha} C \gamma_5 \psi_{\beta j} \rangle = \sum_{I=1}^3 \Delta_I(\mathbf{r}) \epsilon^{\alpha\beta I} \epsilon_{ijI}$$

$\Delta_I(\mathbf{r}) = \Delta_I \exp(2i\mathbf{q}_I \cdot \mathbf{r})$

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Electrical neutrality

B-equilibrium

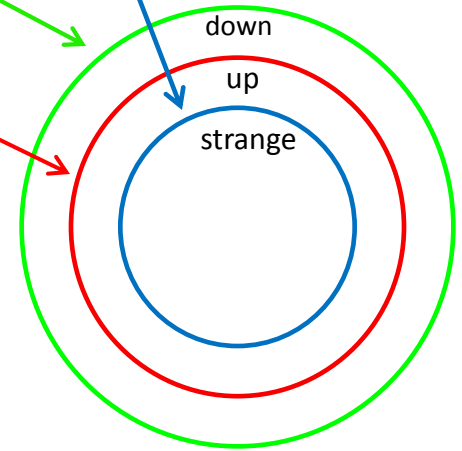
Strange quark mass

$$\mu_u = \mu - \frac{2}{3}\mu_e, \quad \mu_d = \mu + \frac{1}{3}\mu_e, \quad \mu_s = \mu + \frac{1}{3}\mu_e - \frac{M_s^2}{2\mu}$$

**Ginzburg-Landau expansion:**

$$\Omega_{LOFF} = \Omega_{norm} + \sum_{I=1}^3 \left( \frac{\alpha_I}{2} \Delta_I^2 + \frac{\beta_I}{4} \Delta_I^4 + \sum_{J \neq I} \frac{\beta_{IJ}}{4} \Delta_I^2 \Delta_J^2 \right) + O(\Delta^6)$$

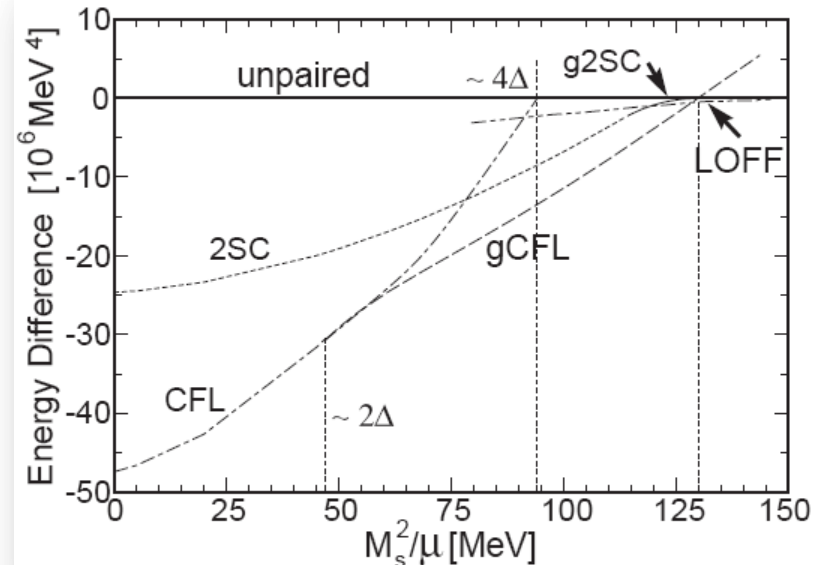
$$\Omega_{norm} = -\frac{3}{12\pi^2} (\mu_u^4 + \mu_d^4 + \mu_s^4) - \frac{\mu_e^4}{12\pi^2}$$



**Results:**

The LOFF phase with three flavors, in the two-plane-wave (2PW) configuration, is favored in comparison with any other phase in the range

$$128 \text{ MeV} < \frac{M_s^2}{\mu} < 150 \text{ MeV}$$

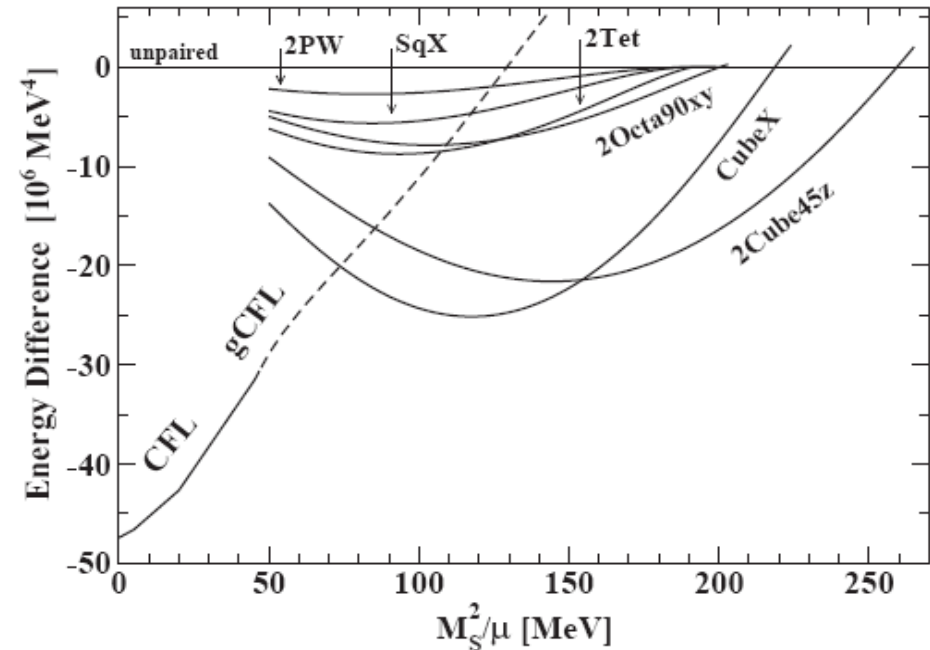
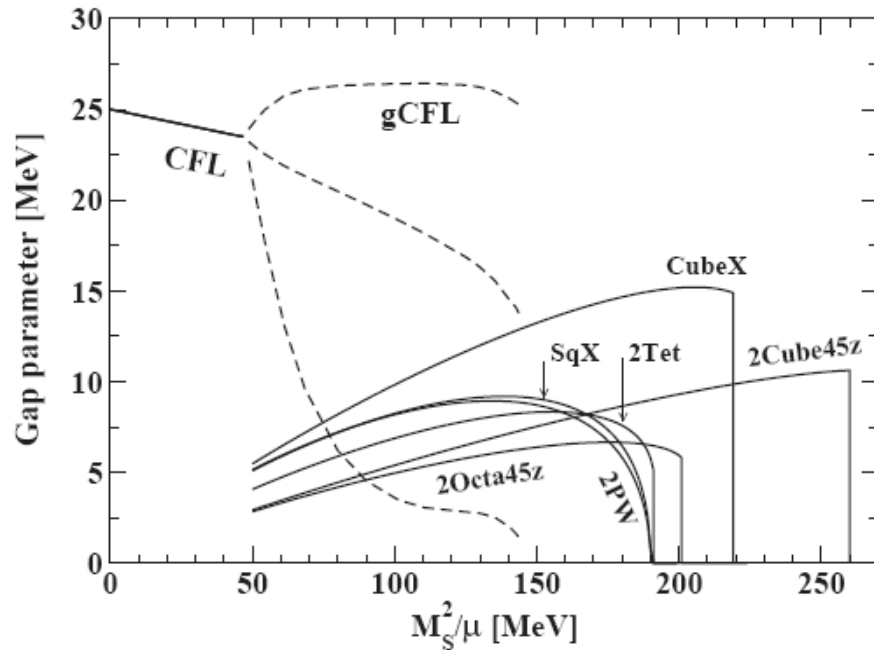


# LOFF phases with more than two plane waves

K.Rajagopal, R.Sharma

Phys. Rev. D **74**, 094019 (2006)

$$\langle \psi_{i\alpha}(x) C \gamma_5 \psi_{j\beta}(x) \rangle \propto \sum_{I=2}^3 \sum_{\mathbf{q}_I^a \in \{\mathbf{q}_I\}} \Delta_I e^{2i\mathbf{q}_I^a \cdot \mathbf{r}} \epsilon_{I\alpha\beta} \epsilon_{Iij}$$



The crystalline LOFF phases named “**CubeX**” and “**2Cube45z**” have gaps and free energies that are comparable to the corresponding parameters in the CFL phase, so they could represent the fundamental state of the matter for wide zones of the QCD phase diagram.

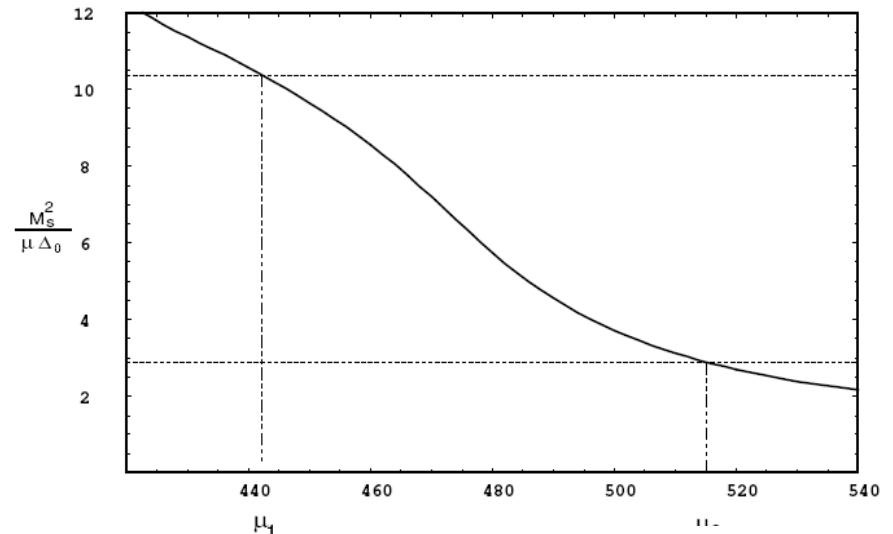
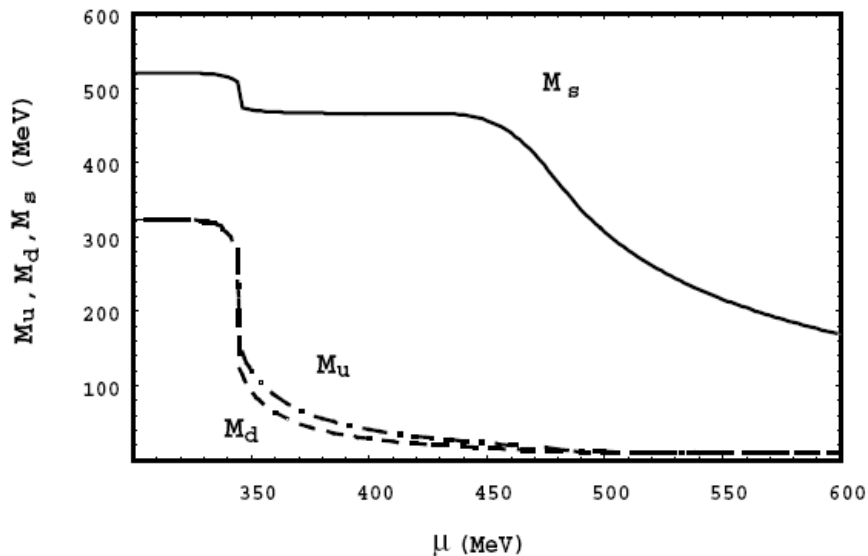
# Quark constituent masses in the LOFF phases

N.D.Ippolito, G.Nardulli, M.Ruggieri,  
JHEP 0704, 036 (2007)

$$\mathcal{L} = \bar{\psi} (i\partial_\mu \gamma^\mu + \hat{\mu}\gamma_0) \psi + \mathcal{L}_{mass} + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_\Delta$$

$$\mathcal{L}_{mass} = - \sum_f m_f \bar{q}_f q_f, \mathcal{L}_4 = G \sum_{a=0}^8 [(\bar{q}\lambda_a q)^2 + (i\bar{q}\gamma_5 \lambda_a q)^2], \mathcal{L}_6 = -K [\det \bar{q}_f (1 + \gamma_5) q_{f'} + \det \bar{q}_f (1 - \gamma_5) q_{f'}]$$

$$\frac{\partial \Omega}{\partial \sigma_f} = 0, \quad \frac{\partial \Omega}{\partial \Delta} = 0, \quad -\frac{\partial \Omega}{\partial \mu_e} = 0 \quad \longrightarrow \quad \sigma_f, \Delta, \mu_e$$



Three flavor LOFF phases favored in comparison with any other phase in the range  $442\text{MeV} < \mu < 515\text{MeV}$

# Three flavor LOFF phases in compact stars

N.D.Ippolito, M.Ruggieri,

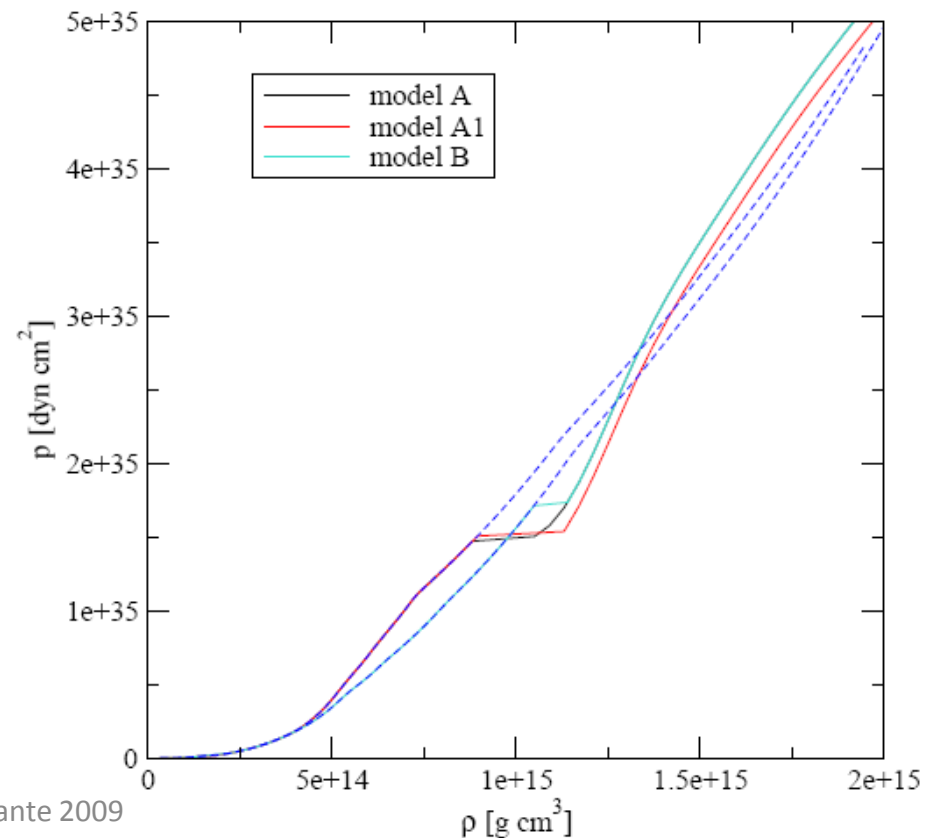
D.H.Rischke(Frankfurt), A.Sedrakian(Frankfurt), F.Weber(San Diego)

Phys. Rev. D **77**, 023004 (2008)

$$P_{LOFF} = -\Omega_{LOFF} \equiv -\Omega_{LOFF} \left( \frac{M_s^2(\mu)}{\mu} \right) \Rightarrow P_{LOFF}(\mu)$$

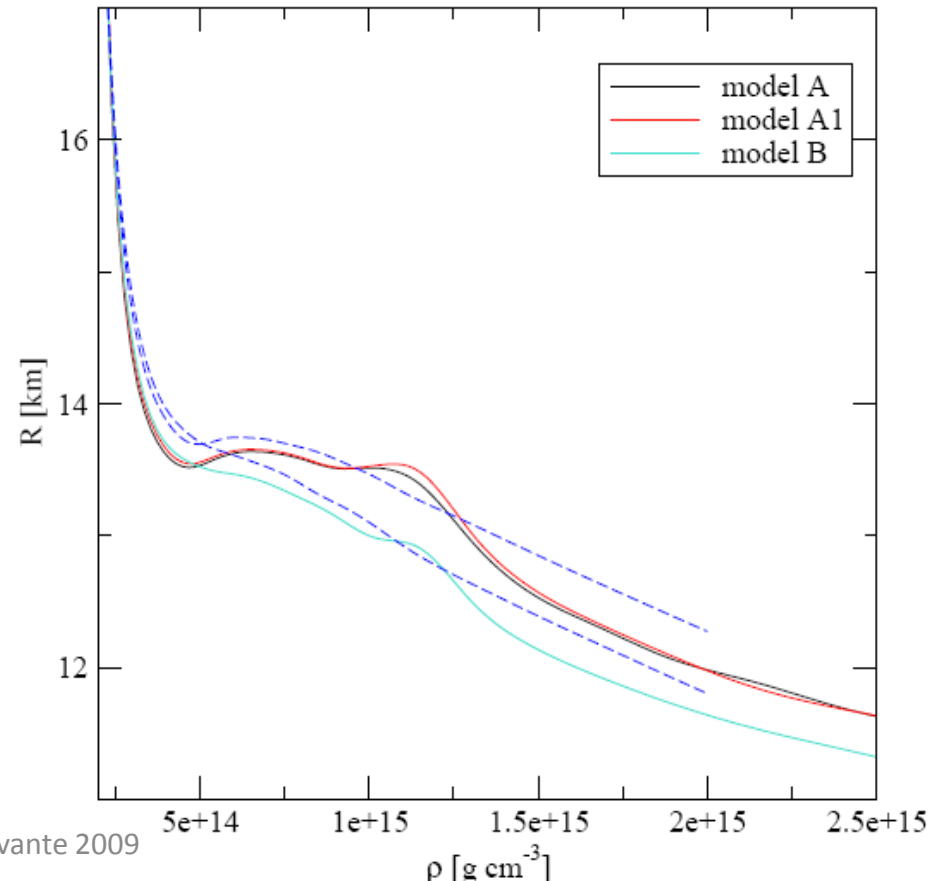
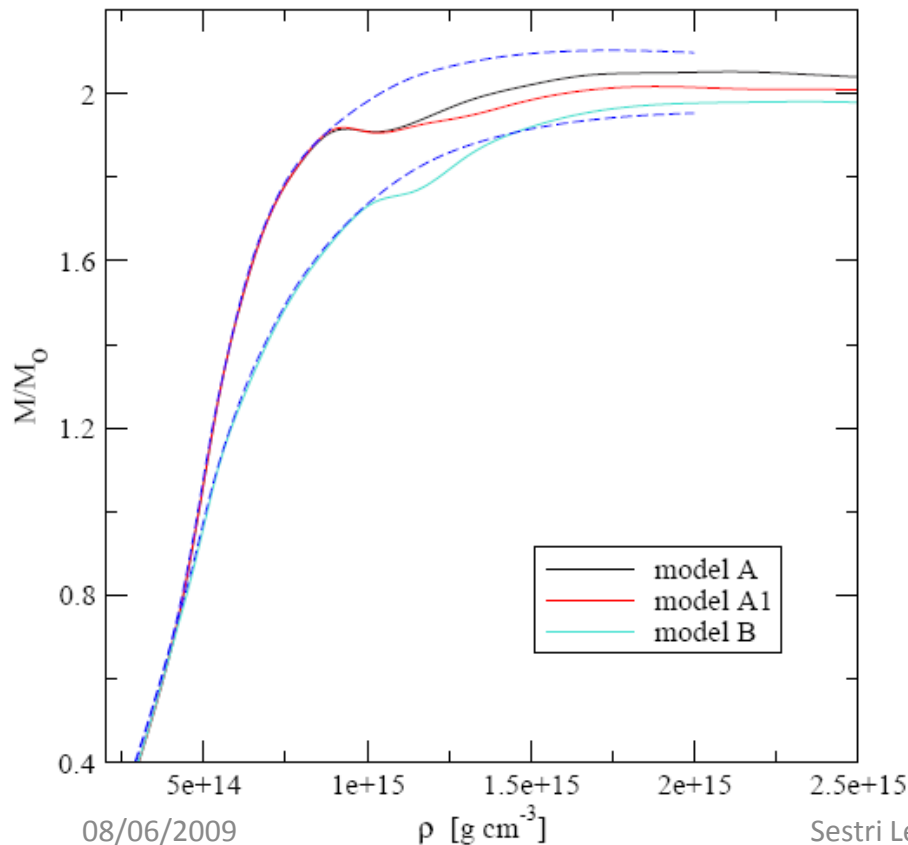
The Equations of State in the figure are obtained with a Maxwell construction (comparison of pressures for a given chemical potential, the phase with the larger pressure being the favored one).

Model A1 is normalized so that the pressure vanishes at zero-density (by adding a bag constant). Models A and B share the same quark EoS, but two different nuclear EoS, and the zero density pressure is shifted by an amount 10 MeV/fm<sup>3</sup>.



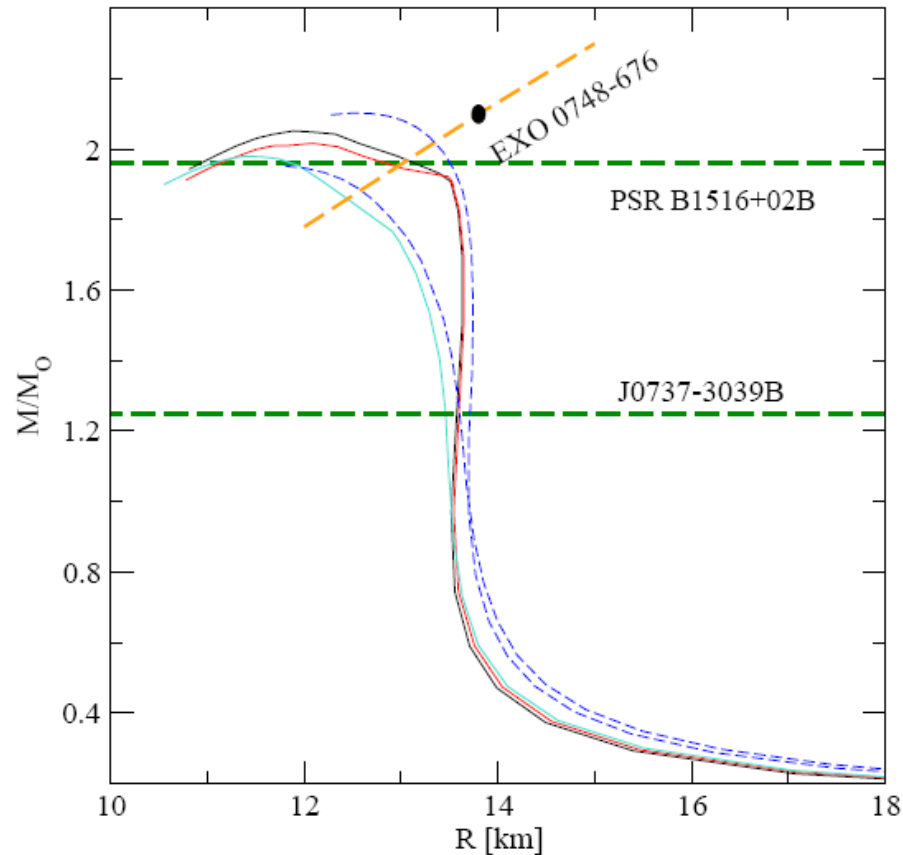
# Results (1)

By putting these three EoS in the Tolman-Oppenheimer-Volkoff (static, spherically symmetrical solutions of Einstein's equations for the gravitational stability of a relativistic body), one obtains the relations between the gross parameters of hybrid stars with LOFF core and nuclear mantle.





## Results (2)



**The three hybrid star models are compatible with the measurements available today for EXO 0748-676, within  $1\sigma$ .**

# *Outlook*

- Neutrino Emissivity and Cooling Curves for hybrid stars with LOFF core;
- Rapidly rotating configurations (already studied), to be compared with more accurate measurements of the frequency of rotation and the moment of inertia;
- Gravitational wave emission from an isolated spinning hybrid star with LOFF matter core.