

On the Dark Matter Halo Mass Function

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CERN

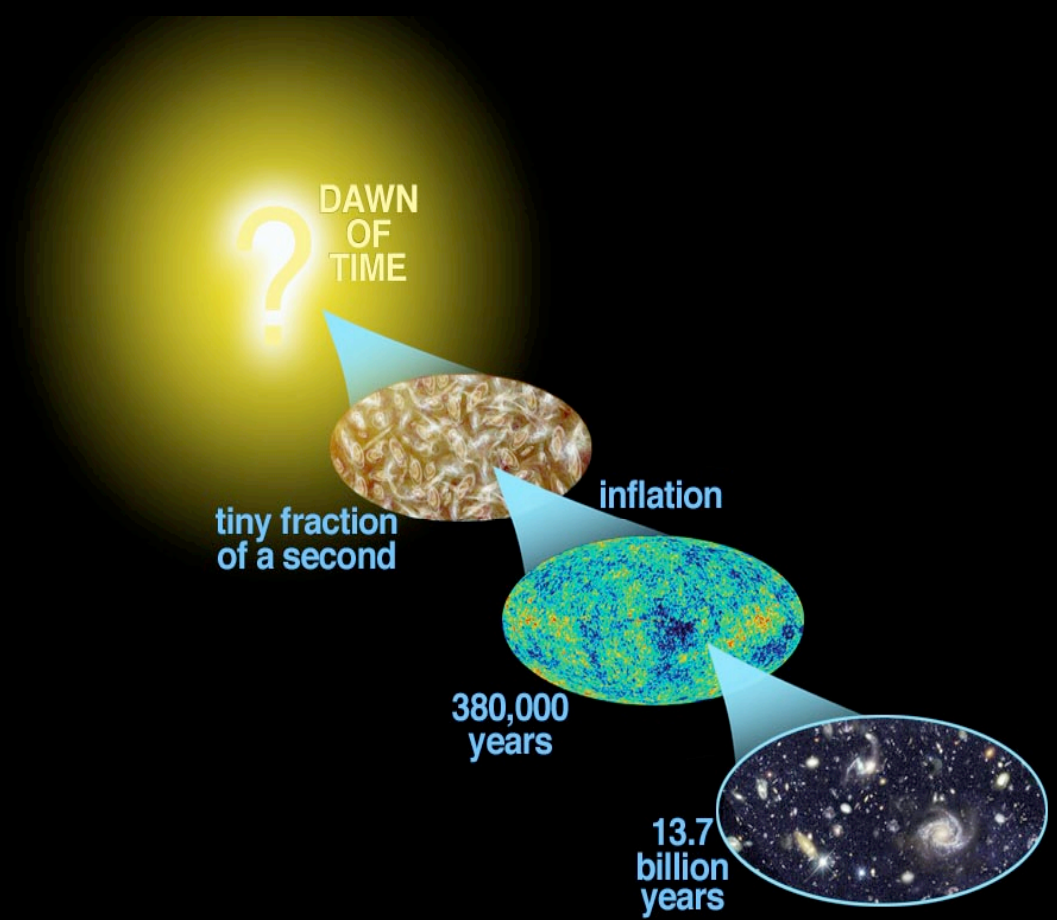


arXiv:0903.1249, 0903.1250, 0903.1251 [astro-ph.CO],
0905.0377 [cond-mat.stat-mech]
with M. Maggiore (University of Geneva)

Plan of the talk

- Introduction to cosmological perturbations and to the dark matter halo mass function
- The excursion set theory
- The halo mass function from first principles for gaussian and non-Gaussian theories
- Conclusions

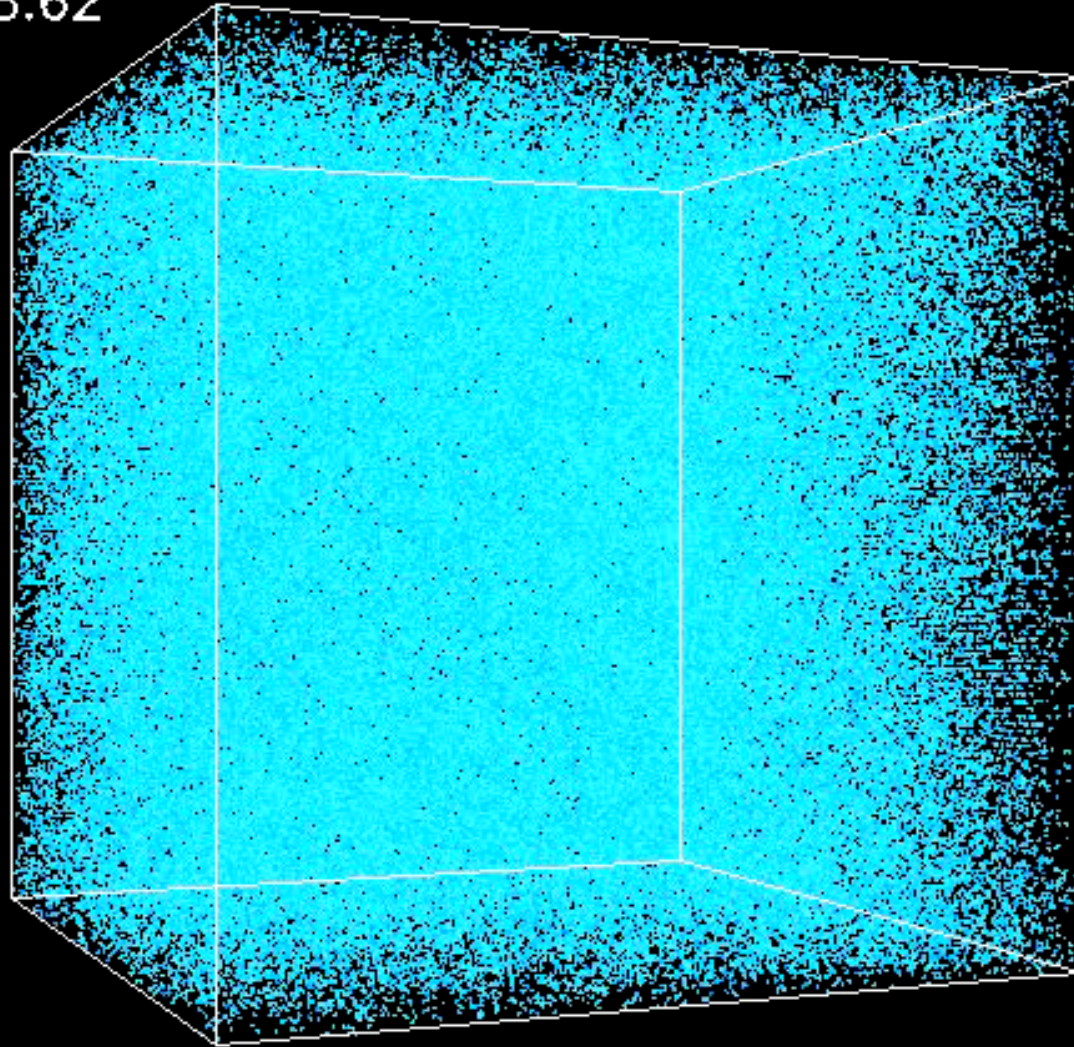
The Inflationary Cosmology



The cornerstones of structure formation

- Initial seeds provided by primordial inflation: spectrum of perturbations nearly flat and nearly gaussian
- Density perturbations grow because of the gravitational instability. The density contrast grows like the scale factor at the linear level
- In the CDM scenario, the first objects to collapse and form dark matter haloes are of low mass
- Hierarchical growth: halo that exists at a given time will have been constructed by the merging of smaller fragments over time
- When haloes merge, their cores survive as distinct subhaloes for some time. In group/cluster scale haloes, these will mark the locations of the galaxies

$Z=28.62$



The Millenium Simulation Project:

<http://www.mpa-garching.mpg.de/galform/virgo/millennium/>

The power spectrum

Perturbing around the average energy density we may define the density contrast

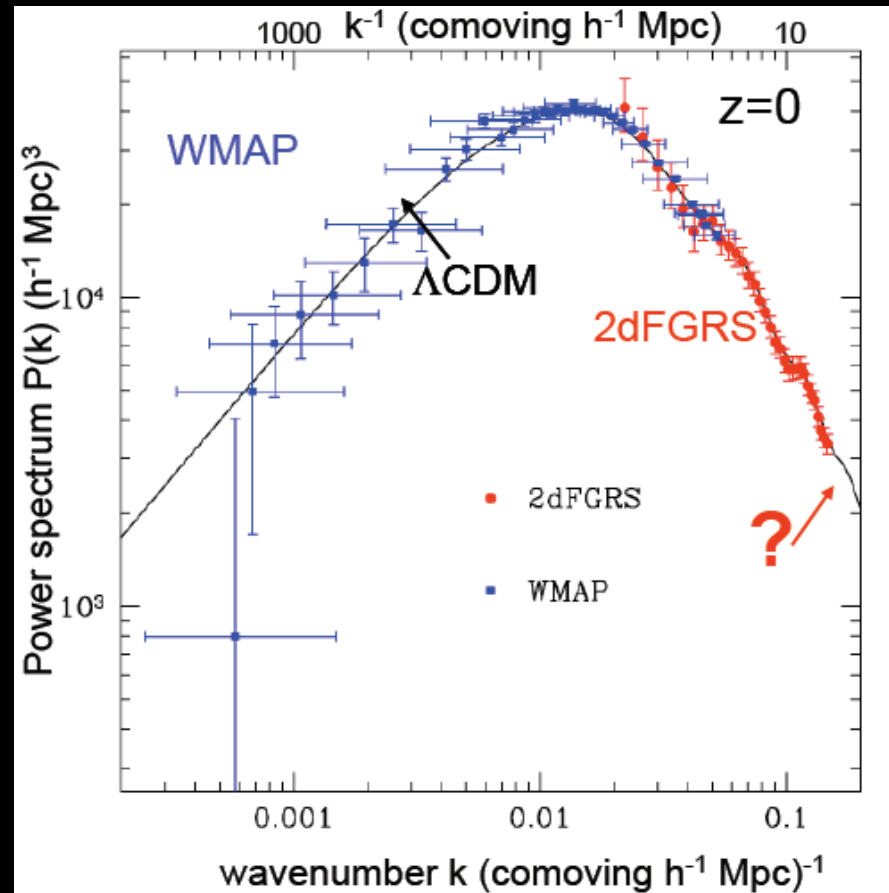
$$\delta(\mathbf{x}, t) \equiv \frac{\rho(\mathbf{x}, t) - \bar{\rho}}{\bar{\rho}} = \int \frac{d^3 k}{(2\pi)^3} \delta_{\mathbf{k}}(t) e^{-i \mathbf{k} \cdot \mathbf{x}}$$

The power spectrum is defined by

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle = (2\pi)^3 P_{\delta}(k) \delta(\mathbf{k} - \mathbf{k}')$$

$$\Delta_{\delta}^2(k) = \frac{k^3 P_{\delta}(k)}{2\pi^2}, \quad P_{\delta} = A k^n T(k)$$

$n \simeq 1$, $T(k)$ = transfer function



$$\ddot{\delta}_{\mathbf{k}} + 2H\dot{\delta}_{\mathbf{k}} = 4\pi G \bar{\rho} \delta_{\mathbf{k}} \Rightarrow \delta_{\mathbf{k}} \propto a$$

$$\delta_{\mathbf{k}} = -\frac{2}{3} \frac{k^2}{\Omega_m H^2} \Phi_{\mathbf{k}}$$

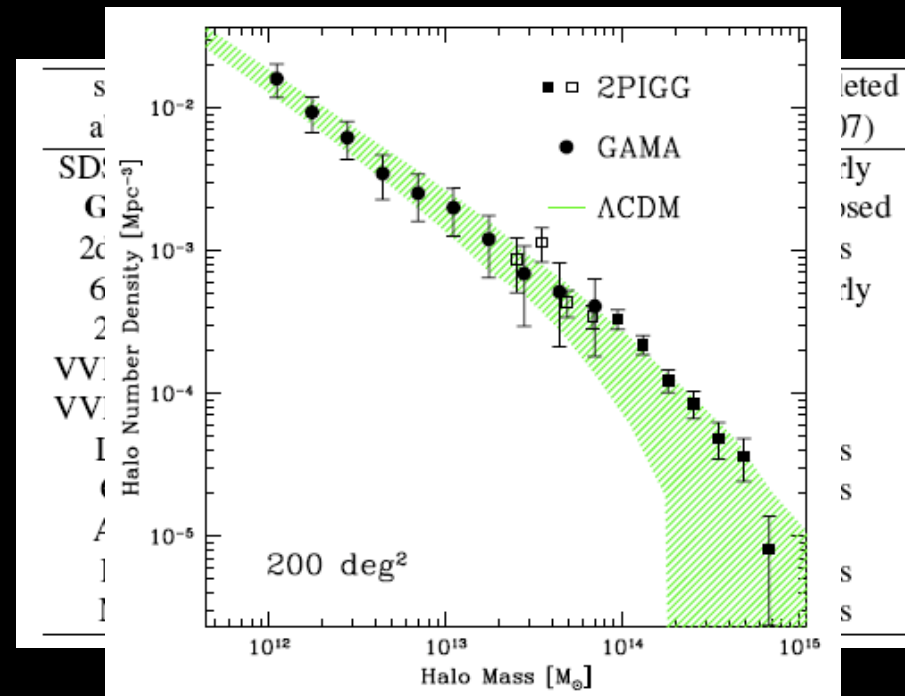
The dark matter halo mass function

A successful theory of structure formation must be able to predict the number density of dark matter haloes as a function of their mass (systems with ~ 200 mean density)

$$\frac{dn}{dM} dM = \frac{\bar{\rho}}{M} \left| \frac{dF}{dM} \right| dM$$

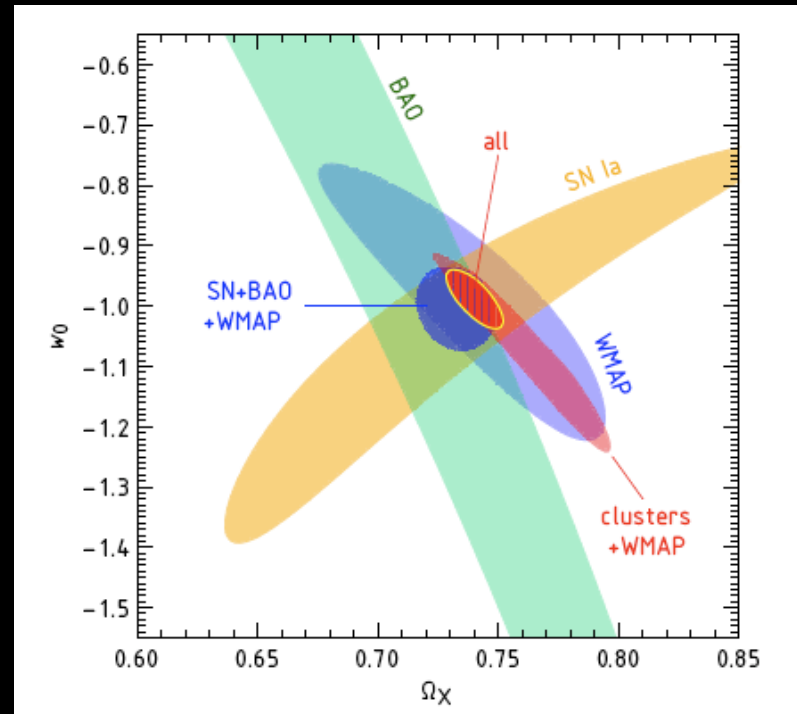
$|dF/dM|$ is the fraction of volume occupied by virialized object of mass between M and $M + dM$

- The DM halo mass function will be accurately tested by planned large-scale galaxy surveys, both ground (e.g. Large Synoptic Survey Telescope, Galaxy And Mass Assembly, volume comparable to horizon size) and satellite (e.g. the ESA EUCLID) based (optical, weak lensing, X-ray emission, SZ effect are complementary)



Why the halo mass function is so relevant ?

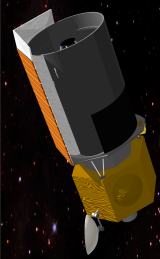
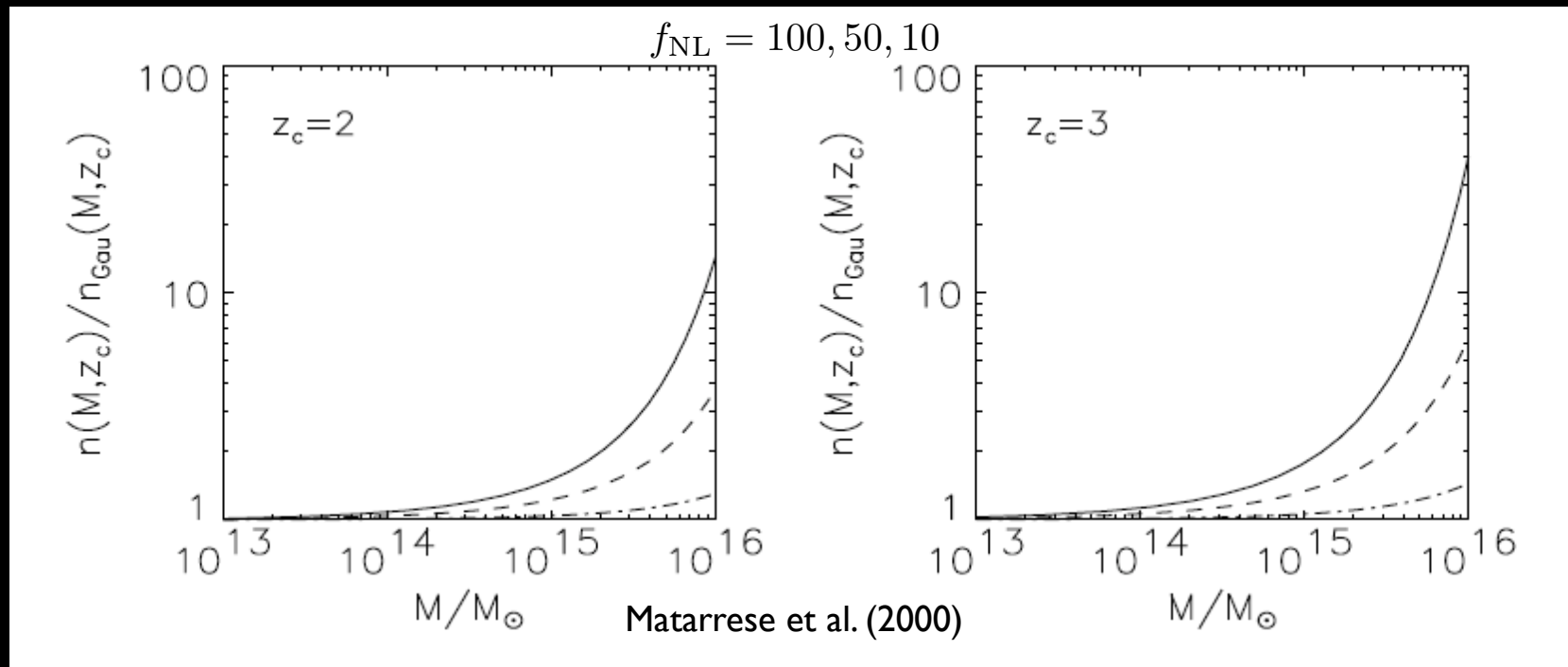
$dn/dV dz$ is exponentially sensitive to the Dark Energy through the growth function



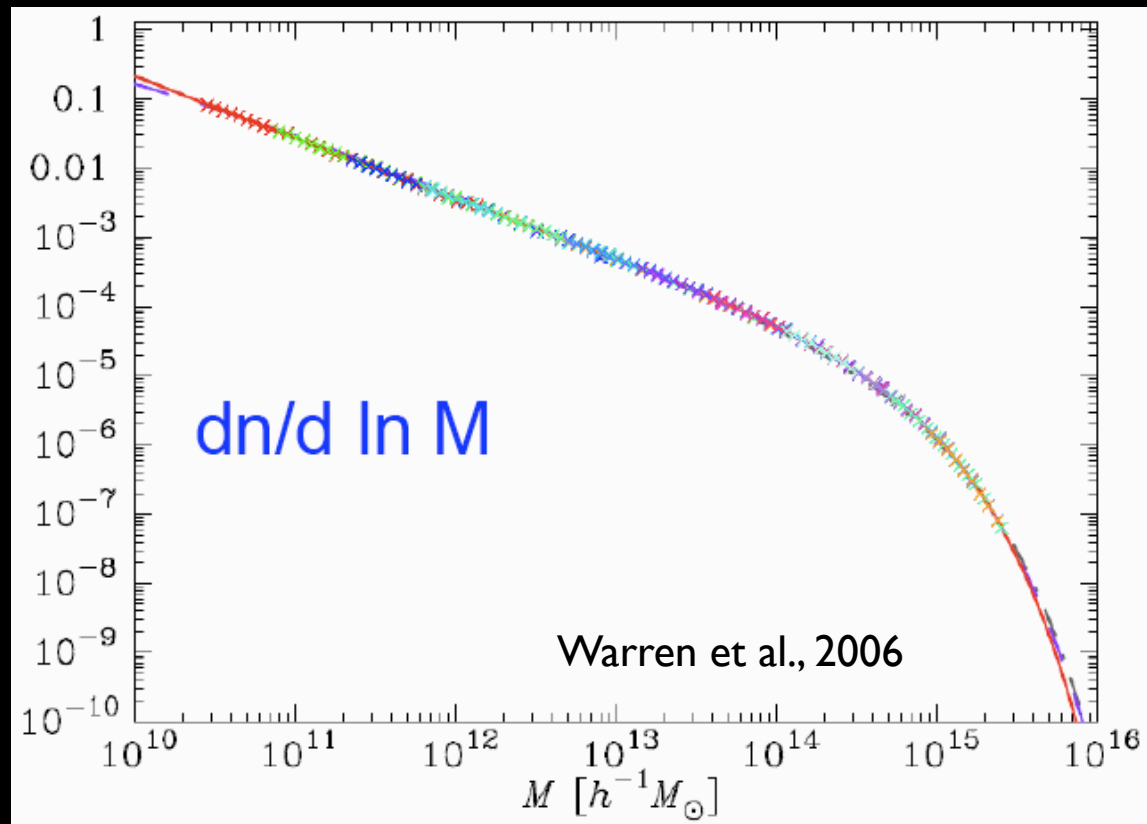
X-ray cluster cosmology white paper, arXiv: 0903.5320

Why the halo mass function is so relevant ?

Rare events are an excellent probe of non-Gaussianity in the primordial power spectrum: $\Phi(x) = \Phi_g(x) + f_{\text{NL}} \Phi_g^2$



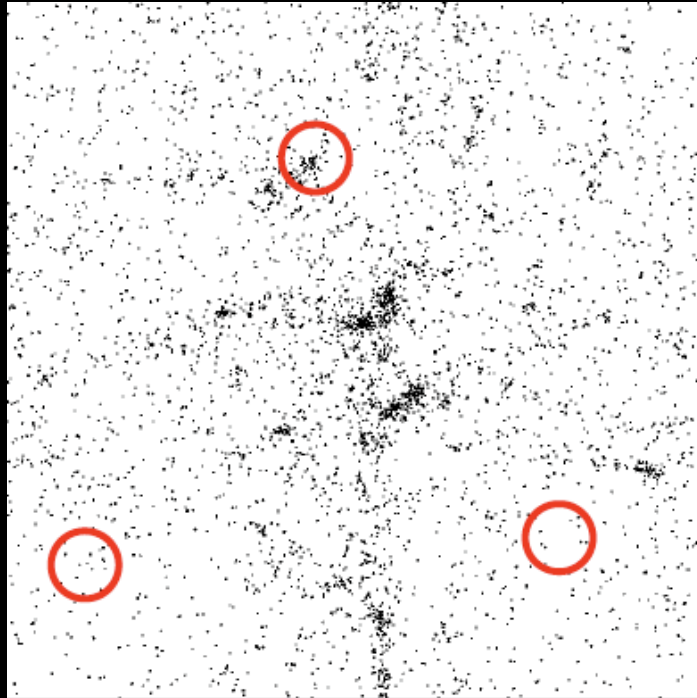
Cosmic Inflation Probe (CIP), a galaxy survey measuring 10 million galaxies at $3 < z < 6$, would offer an opportunity to use this formula to constrain $f_{\text{NL}} \sim 5$ (note that the scale measured by CIP is smaller than that measured by CMB by a factor of $\sim 10!$)



At present the knowledge of the halo mass function comes mainly from N-body simulations

A full theoretical understanding of the halo mass function is still lacking

The smoothing procedure



$$\delta(\mathbf{x}, R) = \int d^3 x' W(|\mathbf{x} - \mathbf{x}'|, R) \delta(\mathbf{x}')$$

Smooth out the perturbation on a sphere of radius R

$$S \equiv \sigma^2(R) \equiv \langle \delta^2(\mathbf{x}, R) \rangle = \int_{-\infty}^{\infty} d \ln k \Delta_{\delta}^2(k) |W(k, R)|^2$$

Window function / filter

Top-hat in momentum space

$$W(k, R) = \theta(k_f - k), \quad k_f = R^{-1}$$

One may not identify a well-defined mass

$$V = 12\pi R^3 \int_0^\infty dx \left(\frac{\sin x}{x} - \cos x \right) \text{ is not defined}$$

Window function / filter

Top-hat in real space

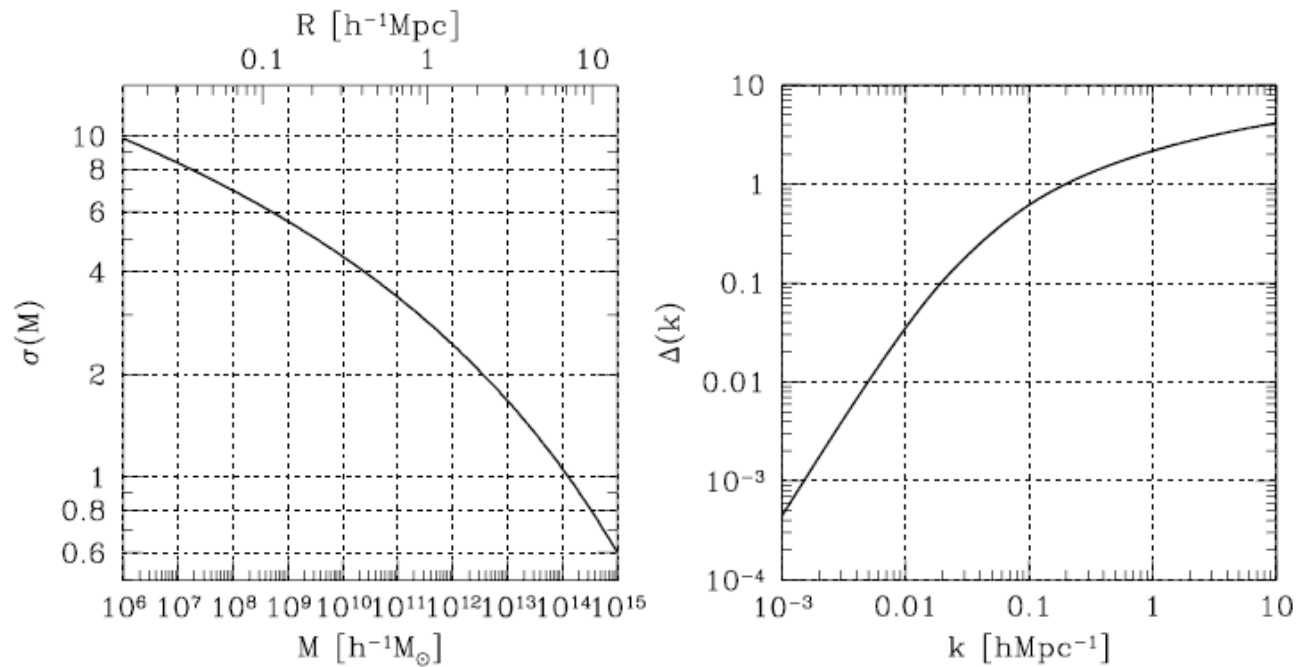
$$W(\mathbf{x}, R) = \frac{3}{4\pi R^3} \theta(R - r)$$

$$W(k, R) = 3 \frac{(\sin(kR) - kR \cos(kR))}{(kR)^3}$$

$$M = \bar{\rho} V, \quad V = \frac{4\pi R^3}{3}$$



N-body simulations use
this window function



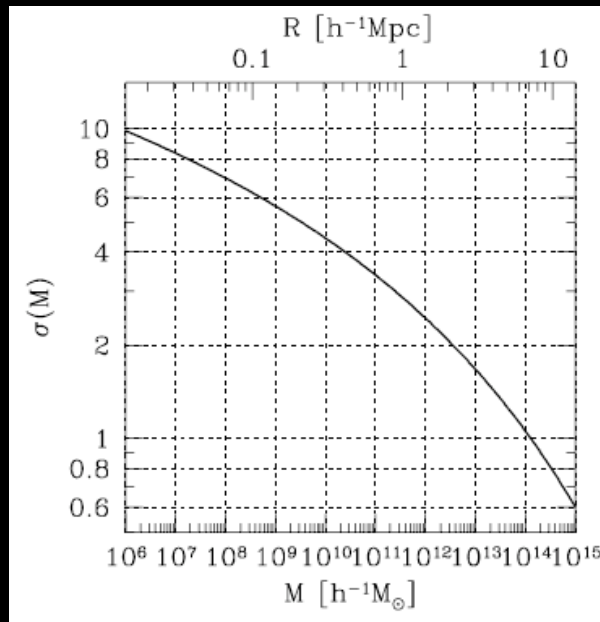
$$\frac{dn}{dM} = 2 \frac{\bar{\rho}}{M^2} f(\sigma) \frac{d \ln \sigma^{-1}}{d \ln M}$$

$$S = \sigma^2(M), \quad f(\sigma) = 2 \sigma^2 \frac{dF}{dS}$$

Dictionary

R

$M(R)$



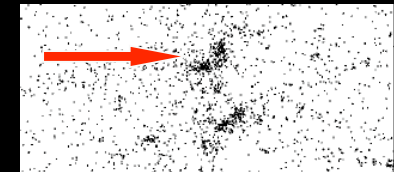
S

$\sigma^2(R)$

Press-Schechter theory (1974)

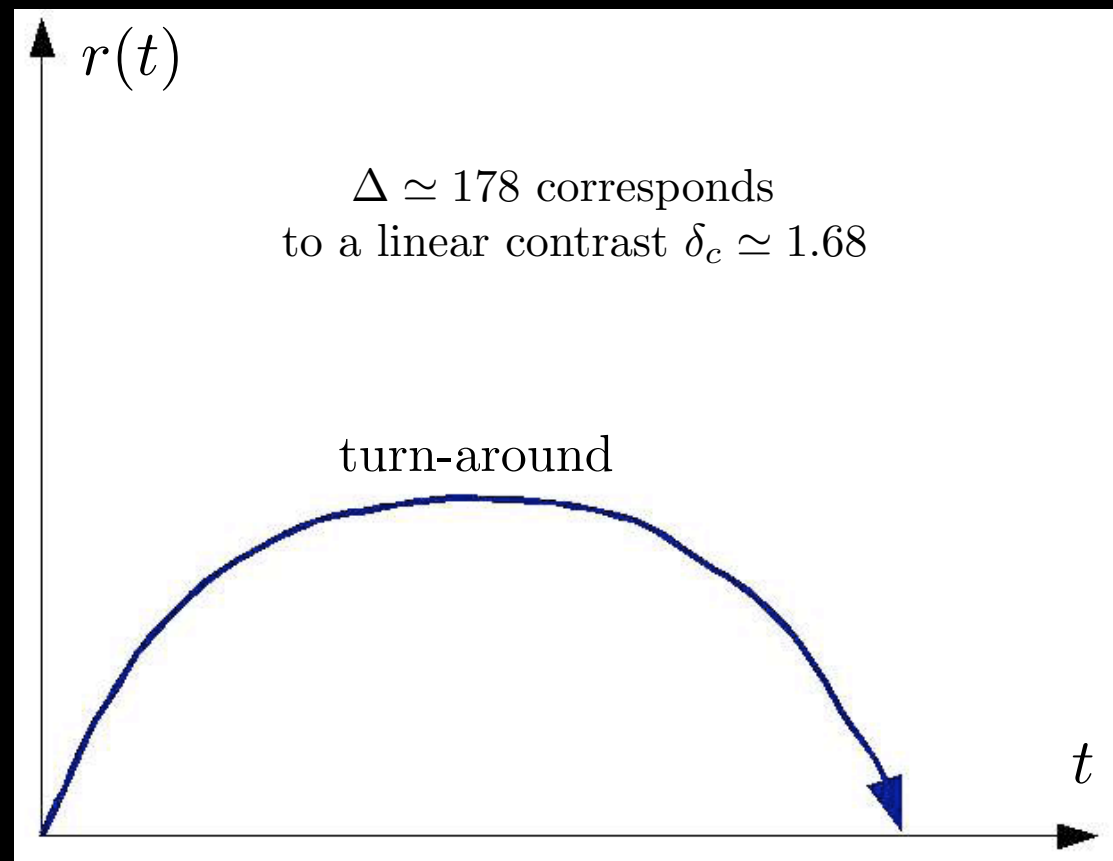
Deal with the clustering problem in two steps:

- Identify the preferential sites for halo formation in Lagrangian space: at any given cosmic time haloes will form preferably in those regions where the initial linear density field is larger than some critical value
- Provide a theoretical framework to follow the non-linear dynamics of the halo distribution responsible for the actual Eulerian distribution of matter clumps as they form by the hierarchical processes of merging. Negligible effects of low-mass haloes on large-mass haloes



The spherical collapse model

According to Birkhoff's theorem, a spherical density overdense perturbation departs from the background evolution and behaves in exactly the same way as part of a closed Universe



Press-Schechter theory and the collapse barrier

- It is assumed that initial linear perturbations are gaussian distributed:

$$\Pi_{\text{PS}} = \frac{1}{\sqrt{2\pi S}} e^{-\delta^2/(2S)}$$

- Virialized objects at a given radius form if the density contrast is larger than the collapse barrier δ_c

$$F_{\text{PS}}(R) = \int_{\delta_c}^{\infty} d\delta \Pi_{\text{PS}}(\delta, S(R)) = \frac{1}{2} \text{Erfc} \left(\frac{\nu(R)}{\sqrt{2}} \right)$$

$$\delta_c = 1.68(1+z) \rightarrow 1.68 D(z) \text{ for } \Lambda\text{CDM}$$

$$\nu = \delta_c / \sigma(R)$$

Cloud-in-cloud problem

In the hierarchical models, the variance $\sigma^2(R)$ diverges at small radii: all mass in the Universe must be finally contained in virialized objects:

$$F(R = 0) = 1$$

instead

$$F_{\text{PS}}(R = 0) = 1/2$$

The PS procedure misses the cases in which, on a given smoothing scale R , the smoothed density contrast $\delta(R)$ is below threshold, but still it happened to be above threshold at some scale $R' > R$. The missing factor of two is put by hand

The excursion set method

Bond, Cole, Efsthathiou, Kaiser (1991)

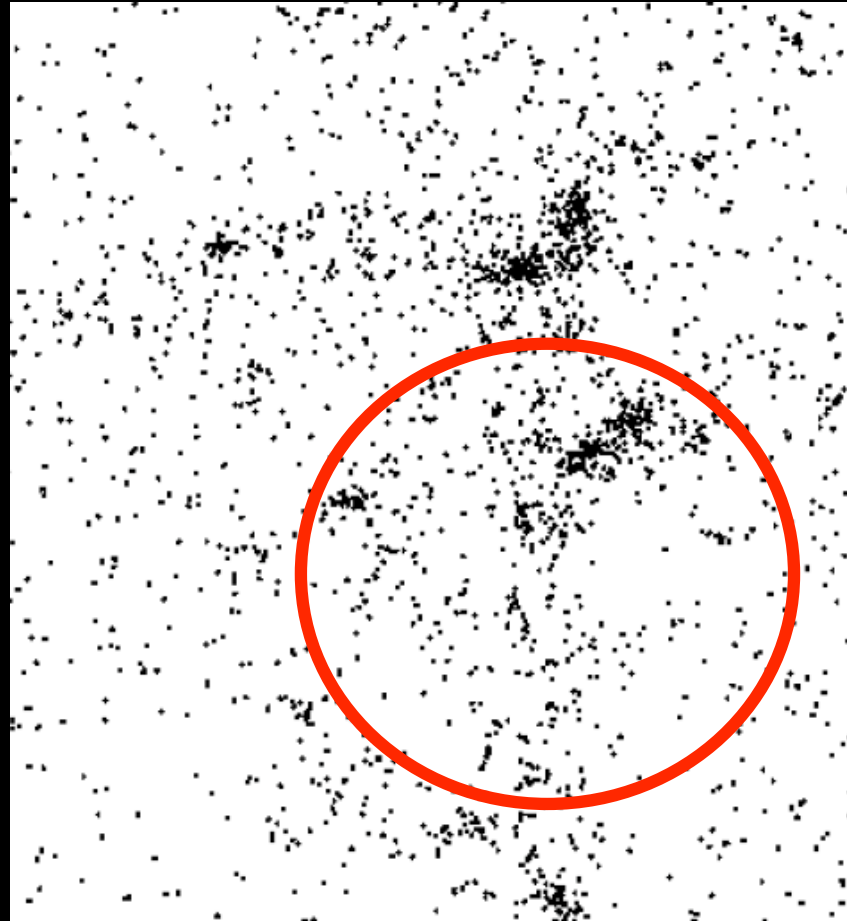
The smoothed density contrast performs a random walk

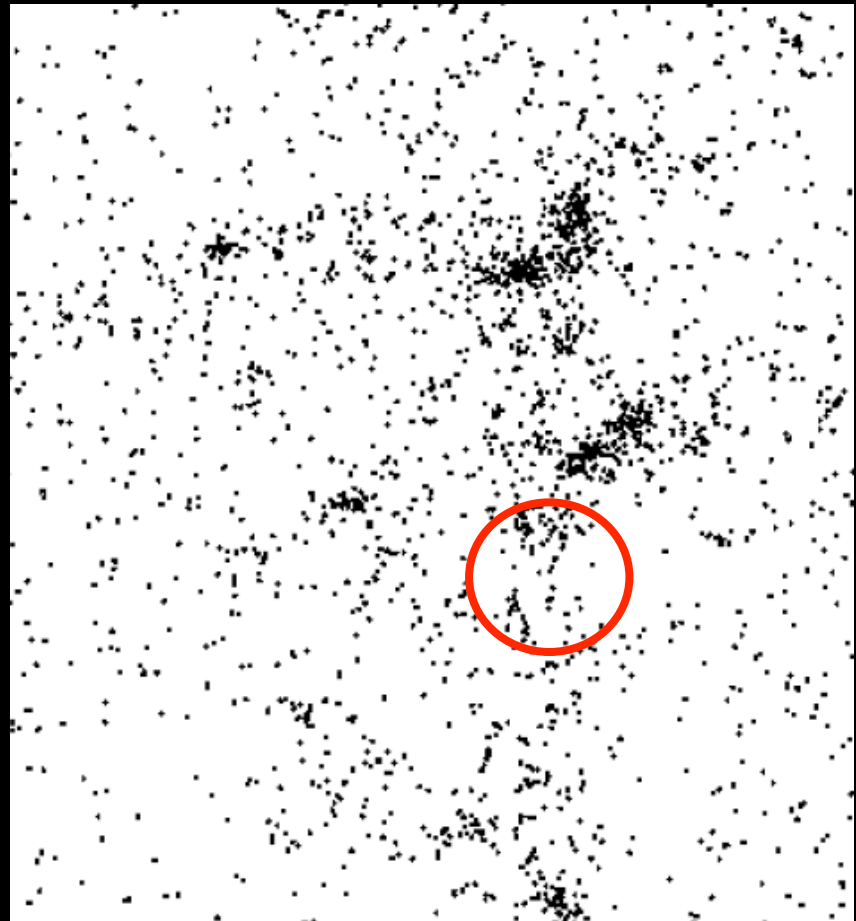
$$\delta(R) = \int \frac{d^3 k}{(2\pi)^3} \delta_{\mathbf{k}} W(k, R) e^{-i\mathbf{k}\cdot\mathbf{x}}$$
$$W(k, R) = \theta(R^{-1} - k)$$

$$\frac{\partial \delta(R)}{\partial R} = \int \frac{d^3 k}{(2\pi)^3} \delta_{\mathbf{k}} \frac{\partial W(k, R)}{\partial R} e^{-i\mathbf{k}\cdot\mathbf{x}}$$
$$= R^2 \int \frac{d^3 k}{(2\pi)^3} \delta_{\mathbf{k}} \delta_D(R - k^{-1}) e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$\left\langle \frac{\partial \delta(R_1)}{\partial R_1} \frac{\partial \delta(R_2)}{\partial R_2} \right\rangle = f(R_1) \delta_D(R_1 - R_2)$$

$$\langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle = (2\pi)^3 P_\delta(k) \delta(\mathbf{k} - \mathbf{k}')$$



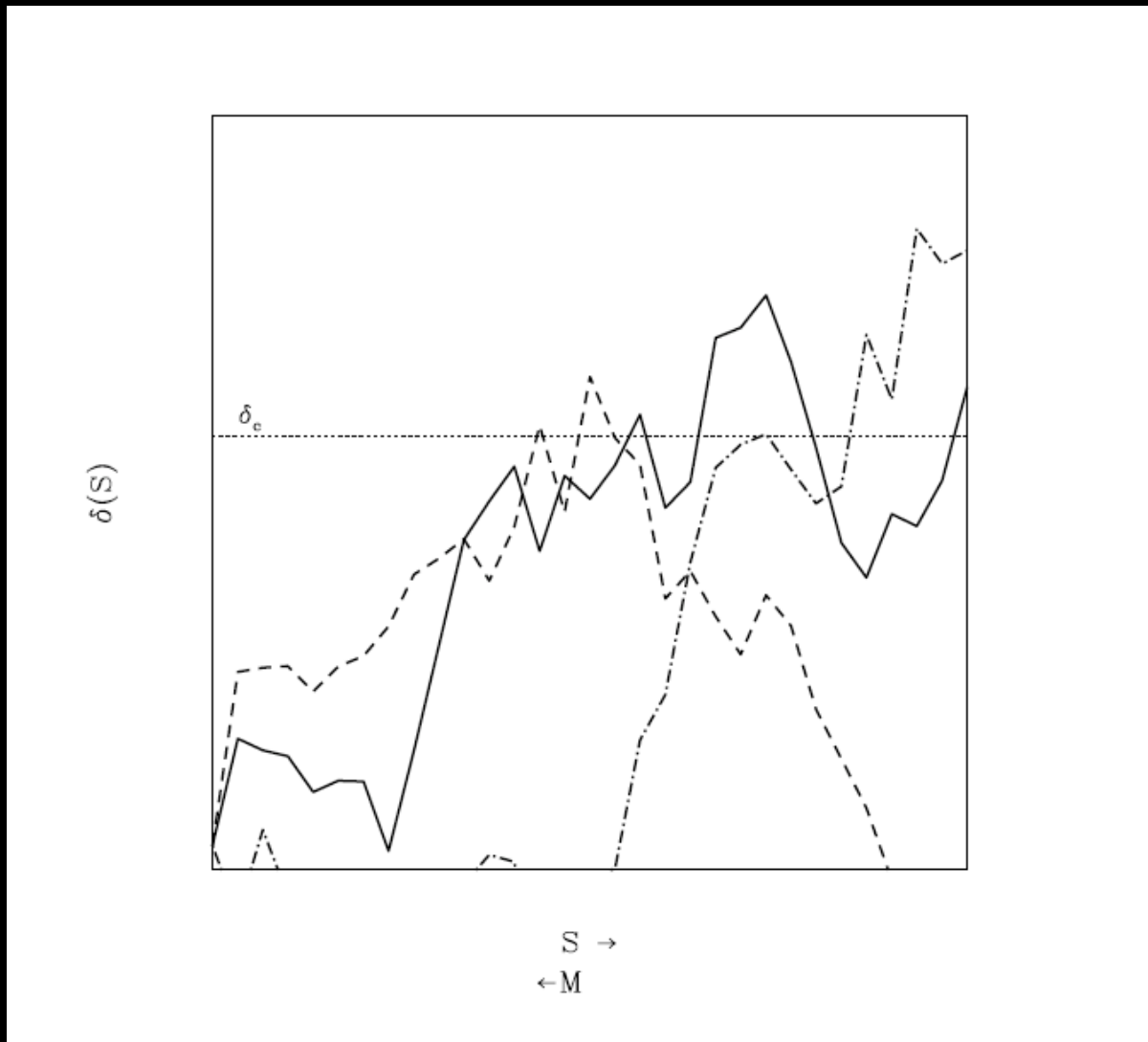


The smoothed density contrast performs a random walk
as a function of the pseudo-time S

MARKOVIAN DYNAMICS & NO MEMORY EFFECTS:
the conditional probability depends on the latest step

$$\frac{\partial \delta(S)}{\partial S} = \eta(S)$$
$$\langle \eta(S_1) \eta(S_2) \rangle = \delta(S_1 - S_2)$$

$$S \equiv \sigma^2(R) \equiv \langle \delta^2(\mathbf{x}, R) \rangle = \int_{-\infty}^{\infty} d \ln k \Delta_{\delta}^2(k) |W(k, R)|^2$$



The normalization of the PS theory is not correct because it does not discard multiple crossings

The problem of finding the probability of halo formation can be matched into the so-called

FIRST-PASSAGE TIME PROBLEM

find the probability that a particle subject to a random walk passes for the first time through a given point

Very well-known problem for markovian dynamics;
application in chemical kinetics, biology, etc.,
for a textbook, see Redner, 2001

A markovian random walk
with diffusion coefficient D :

$$\langle \delta^2(S) \rangle = D S$$

satisfies a Fokker-Planck (diffusion) equation

$$\frac{\partial \Pi}{\partial S} = \frac{D}{2} \frac{\partial^2 \Pi}{\partial \delta^2}$$

with boundary conditions:

$$\delta(S = 0) = \delta_D(\delta)$$

$$\Pi(\delta_c, S) = 0 \text{ (absorbing barrier)}$$

The probability is given by

$$\Pi(\delta, S) = \frac{1}{\sqrt{2\pi S}} \left(e^{-\delta^2/(2DS)} - e^{-(2\delta_c - \delta)^2/(2DS)} \right)$$

The first-passage time probability is inferred from the survival probability

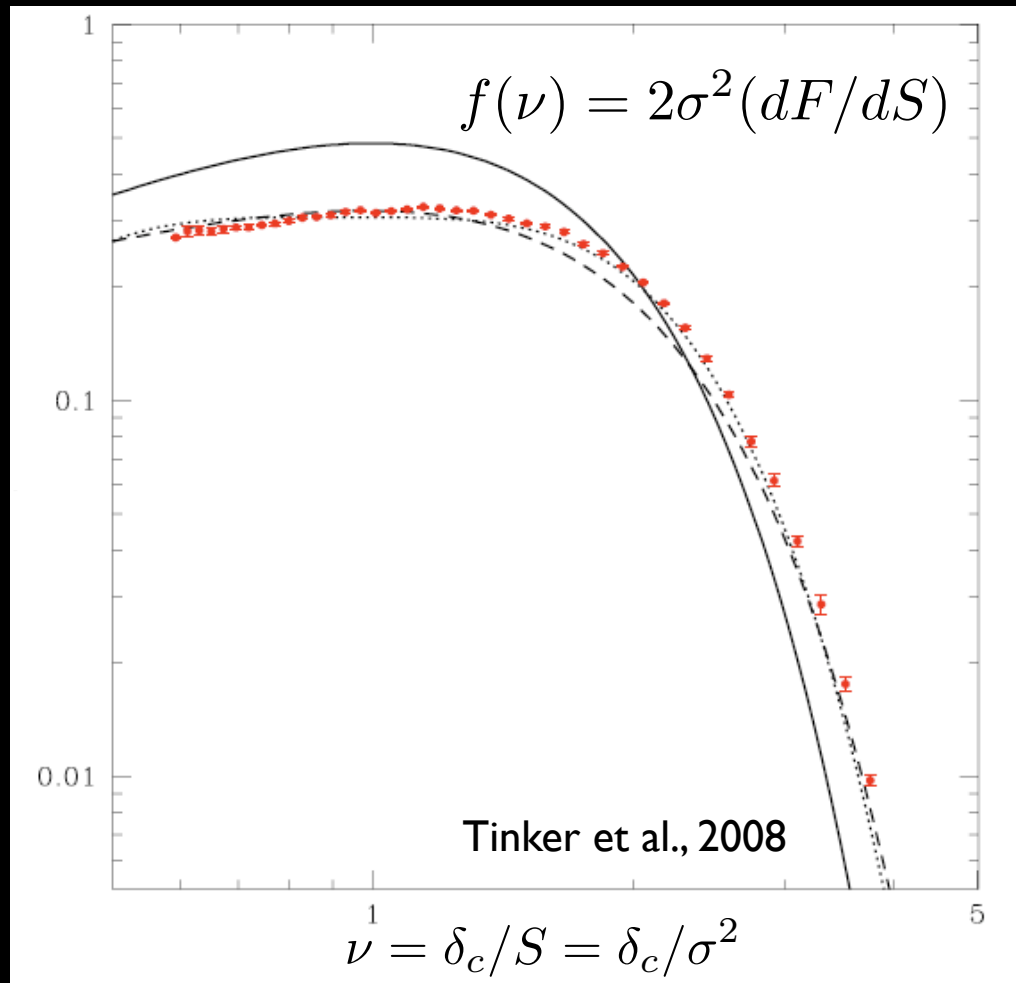
$$\int_{-\infty}^{\delta_c} d\delta \Pi(\delta, S) = 1 - F(S)$$



$$D = 1$$

$$\frac{dF}{dS} = - \int_{-\infty}^{\delta_c} d\delta \frac{\partial \Pi}{\partial S} = \frac{2}{\sqrt{2\pi S^3/2}} e^{-\delta_c^2/(2S)}$$

The PS prediction is recovered with the missing factor of two



At large masses, the dark matter halo mass function is overestimated by a factor ~ 10 ; at small halo masses it overestimates it by a factor ~ 2

Some comments

- The excursion set method correctly is **not** in agreement with the N-body results
- As formulated, it does hold only for a top-hat filter in momentum space, which is **not** used in N-body simulations
- Smoothing with the top-hat filter in real space and/or dealing with non-Gaussianities makes the various random steps **correlated**: the dynamics is non-markovian and memory effects are introduced

Window function in real space

$$W(\mathbf{x}, R) = \frac{3}{4\pi R^3} \theta(R - r)$$

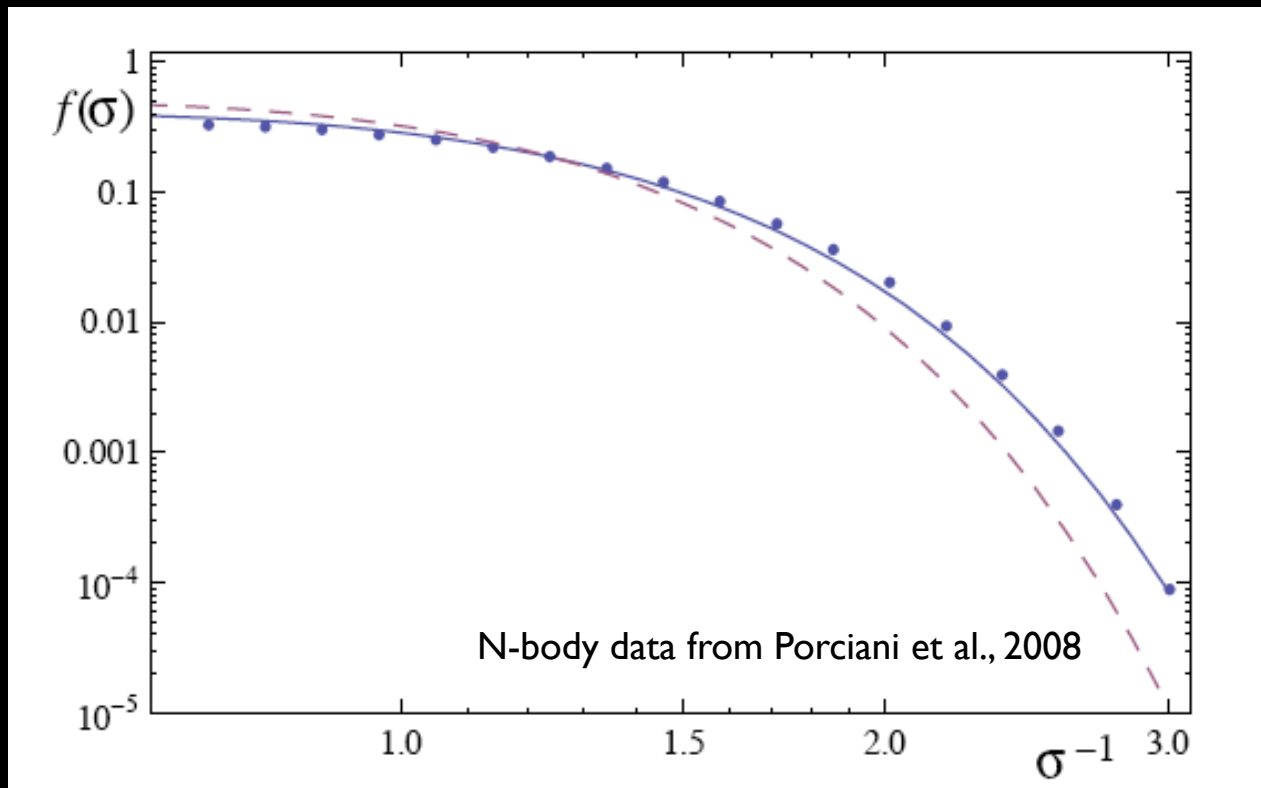
$$W(k, R) = 3 \frac{(\sin(kR) - kR \cos(kR))}{(kR)^3}$$

$$\delta(R) = \int \frac{d^3 k}{(2\pi)^3} \delta_k \widetilde{W}(k, R) e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$\frac{\partial \delta(S)}{\partial S} = \eta(S)$$
$$\langle \eta(S_1) \eta(S_2) \rangle \neq \delta(S_1 - S_2)$$

$$\langle \delta(S_1) \delta(S_2) \rangle = \int_0^{S_1} dS'_1 \int_0^{S_2} dS'_2 \langle \eta(S'_1) \eta(S'_2) \rangle$$

- The excursion set method results may not be trusted unless the correlation among the random walks (non-markovian effects) introduced by the top-hat filter in real space and/or NG are accounted for
- One has to find an explanation of why on large halo masses the exponential decay is slower: is it that the collapse barrier is smaller or something else?



No ad hoc parameters

First-passage problem with non-markovian dynamics

- Non-Markov is the rule, Markov is the exception (van Kampen, 1998)
- Non-markovian dynamics (random walk steps correlated) introduces memory effects
- The probability does not satisfy a simple diffusion Fokker-Planck equation. In fact, the equation is not even known in most cases
- Memory effects imposes a discretization of the problem for which the definition of the collapse barrier is not a priori obvious: a given stochastic trajectory may exit a given domain by jumping over the boundary without hitting it. Particular care has to be devoted to the passage from the discrete to the continuum

van Kampen and Oppenheim, 1972;
Hanggi et al., 1981;
Weiss et al., 1983; Knessl, 1986;

Path integral approach

1) Suppose our smoothed density contrast moves stochastically in time. We indicate it by $\xi(S)$

2) We discretize the trajectory in infinitesimal steps

$$S_k = k\epsilon, k = 1, \dots, n, n = S/\epsilon$$

3) The probability density in the space of trajectories is

$$W(\delta_0; \delta_1, \dots, \delta_n; S_n) \equiv \langle \delta_D(\xi(S_1) - \delta_1) \cdots \delta_D(\xi(S_n) - \delta_n) \rangle$$

4) The probability to reach a given point at a given time is therefore

$$\Pi_\epsilon(\delta_0; \delta_n, S_n) = \int_{-\infty}^{\delta_c} d\delta_1 \cdots \int_{-\infty}^{\delta_c} d\delta_{n-1} W(\delta_0; \delta_1, \dots, \delta_{n-1}, \delta_n; S_n)$$

5) We can express the probability in terms of the correlators of the theory

$$\begin{aligned} \Pi_\epsilon(\delta_0; \delta_n, S_n) &= \int_{-\infty}^{\delta_c} \delta_1 \cdots \delta_{n-1} \int \frac{d\lambda_1}{2\pi} \cdots \frac{d\lambda_n}{2\pi} \\ &\times e^{\left\{ i \sum_{i=1}^n \lambda_i \delta_i + \sum_{p=2}^{\infty} \sum_{i_1=1}^n \cdots \sum_{i_p=1}^n \lambda_{i_1} \cdots \lambda_{i_p} \langle \xi_{i_1} \cdots \xi_{i_p} \rangle_c \right\}} \end{aligned}$$

6) For the markovian case $\langle \xi(S_i) \xi(S_j) \rangle_c = \min(S_i, S_j)$ and

$$\begin{aligned} W^{\text{gau}}(\delta_0; \delta_1, \cdots, \delta_n, S_n) &= \frac{1}{(2\pi\epsilon)^{n/2}} \exp \left\{ -\frac{1}{2\epsilon} \sum_{i=0}^{n-1} (\delta_{i+1} - \delta_i)^2 \right\} \\ &= \Psi_\epsilon(\delta_n - \delta_{n-1}) W^{\text{gau}}(\delta_0; \delta_1, \cdots, \delta_{n-1}, S_{n-1}) \\ \Psi_\epsilon(\delta_n - \delta_{n-1}) &= \frac{1}{(2\pi\epsilon)^{1/2}} \exp \left\{ -(\delta_n - \delta_{n-1})^2 / (2\epsilon) \right\} \end{aligned}$$

The probability does not obey
a simple Fokker-Planck equation

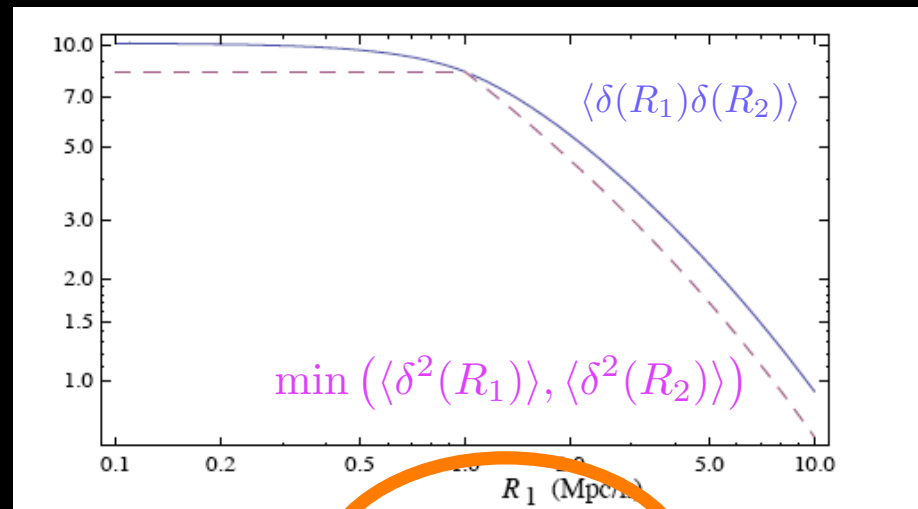
$$\frac{\partial}{\partial S_n} \Pi_\epsilon(\delta_0; \delta_n, S_n) = \frac{1}{2} \frac{\partial^2}{\partial \delta_n^2} \Pi_\epsilon(\delta_0; \delta_n, S_n)$$
$$+ \sum_{k=1}^{n-1} \frac{\partial \langle \xi_k \xi_n \rangle_c}{\partial S_n} \frac{\partial}{\partial \delta_n} \int_{-\infty}^{\delta_c} d\delta_1 \cdots \delta_{n-1} \frac{\partial}{\partial \delta_k} W(\delta_0; \delta_1, \cdots, \delta_n, S_n)$$

memory terms

We take now the correlator induced by a top-hat filter in real space

$$\langle \delta(R_1) \delta(R_2) \rangle = \frac{1}{2\pi^2} \int_0^\infty dk k^2 P_\delta(k) T^2(k) W(k, R_1) W(k, R_2)$$

$$W(k, R) = \frac{3}{4} \frac{\sin(kR) - kR \cos(kR)}{(kR)^3}$$

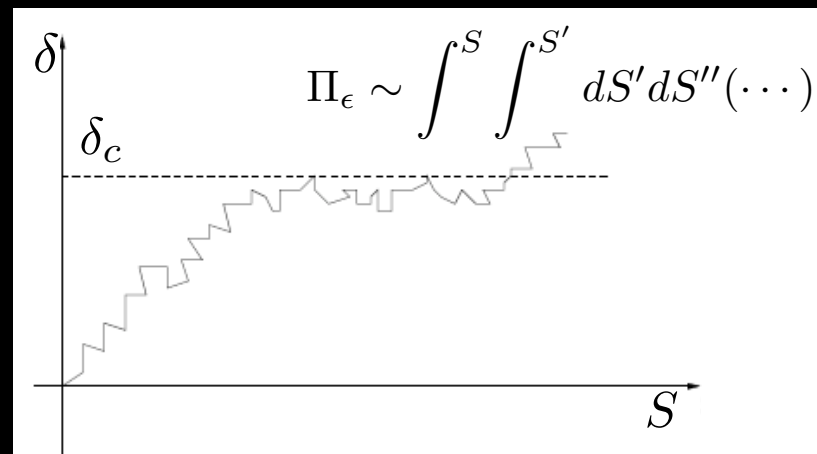
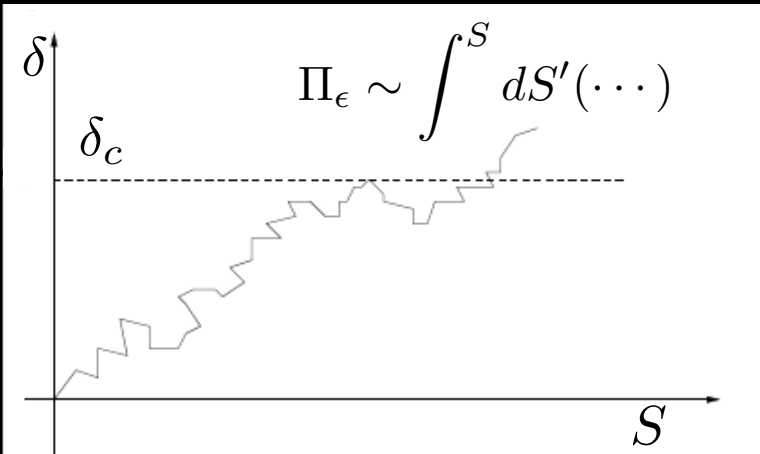


$$\langle \xi(S_i) \xi(S_j) \rangle \simeq \min(S_i, S_j) + \Delta(S_i, S_j)$$

$$\Delta(S_i, S_j) \simeq \kappa \frac{S_i(S_j - S_i)}{S_j}, \quad \kappa \simeq 0.44$$

We expand now the correlator around the markovian contribution and get the non-markovian contributions

$$\begin{aligned}
 \Pi_{\epsilon}^{\text{mem}}(\delta_0; \delta_n; S_n) &= \sum_{i=1}^{n-1} \Delta_{in} \partial_n [\Pi_{\epsilon}^{\text{gau}}(\delta_0; \delta_c; S_i) \\
 &\times \Pi_{\epsilon}^{\text{gau}}(\delta_c; \delta_n; S_n - S_i)] \\
 \Pi_{\epsilon}^{\text{mem-mem}}(\delta_0; \delta_n; S_n) &= \sum_{i < j} \Delta_{ij} \Pi_{\epsilon}^{\text{gau}}(\delta_0; \delta_c; S_i) \\
 &\times \Pi_{\epsilon}^{\text{gau}}(\delta_c; \delta_c; S_j - S_i) \\
 &\times \Pi_{\epsilon}^{\text{gau}}(\delta_c; \delta_n; S_n - S_j)
 \end{aligned}$$



Memory terms introduce discrete sums of correlated quantities:
in order to take the continuum limit, we must know the behaviour
of the probability around the barrier at order $\mathcal{O}(\epsilon)$

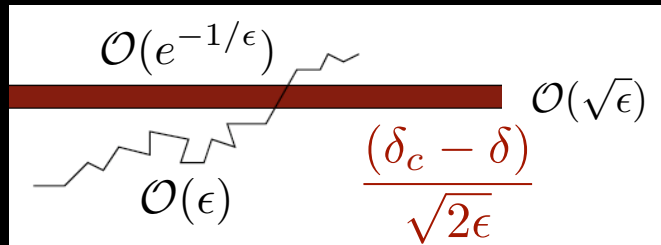
$$\sum_{i=1}^{n-1} \rightarrow \frac{1}{\epsilon} \int_0^{S_n} dS_i$$

$$\begin{aligned} \Pi_{\epsilon}^{\text{mem}}(\delta_0; \delta_n; S_n) &= \partial_n \int_0^{S_n} dS_i \Delta(S_i, S_n) \\ \times \lim_{\epsilon \rightarrow 0} \frac{1}{\epsilon} \Pi_{\epsilon}^{\text{gau}}(\delta_0; \delta_c, S_i) \Pi_{\epsilon}^{\text{gau}}(\delta_c; \delta_n, S_n - S_i) \end{aligned}$$

We need to know how $\Pi_{\epsilon}^{\text{gau}}(\delta_0; \delta_c, S)$ and $\Pi_{\epsilon}^{\text{gau}}(\delta_c; \delta_c, S)$
vanish at the barrier collapse point

The expansion around the barrier

$$\begin{aligned}
 \Pi_\epsilon^{\text{gau}}(\delta_0; \delta, S + \epsilon) &= \Pi_\epsilon^{\text{gau}}(\delta_0; \delta, S) + \epsilon \frac{\partial \Pi_\epsilon^{\text{gau}}}{\partial S} + \frac{\epsilon^2}{2} \frac{\partial^2 \Pi_\epsilon^{\text{gau}}}{\partial S^2} + \dots \\
 &= \int_{\delta - \delta_c}^{\infty} d(\Delta\delta) \Psi_\epsilon(\Delta\delta) \Pi_\epsilon^{\text{gau}}(\delta_0; \delta - \Delta\delta, S) \\
 &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{\partial^n \Pi_\epsilon^{\text{gau}}(\delta_0; \delta, S)}{\partial S^n} \int_{\delta - \delta_c}^{\infty} d(\Delta\delta) (\Delta\delta)^n \Psi_\epsilon(\Delta\delta)
 \end{aligned}$$



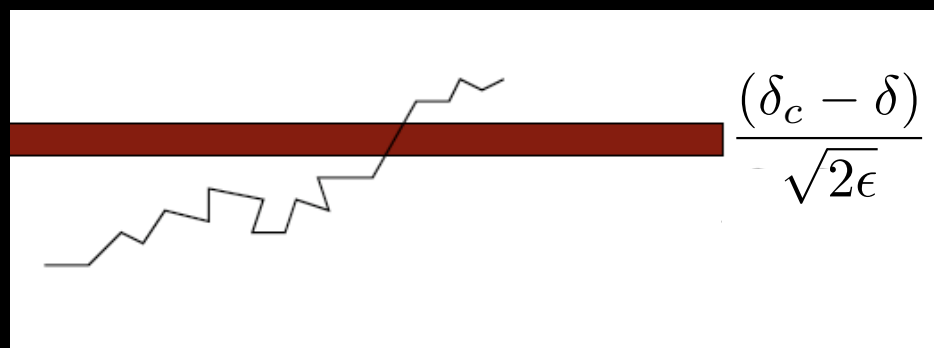
- If $(\delta_c - \delta)/\sqrt{2\epsilon} > 1$, then $\Pi_\epsilon^{\text{gau}} = \Pi_{\epsilon=0}^{\text{gau}} + \epsilon \Pi_{(1)}^{\text{gau}} + \dots$
and $\partial \Pi_{\epsilon=0}^{\text{gau}} / \partial S = 1/2 \partial^2 \Pi_{\epsilon=0}^{\text{gau}} / \partial \delta^2$
- If $(\delta_c - \delta)/\sqrt{2\epsilon} \approx 1$ then $1/2 \Pi_\epsilon^{\text{gau}}(\delta_0; \delta_c, S) = -(\epsilon/2\pi)^{1/2} \partial \Pi_\epsilon^{\text{gau}}(\delta_0; \delta, S) / \partial S |_{\delta=\delta_c} + (\text{same order})$
- If $(\delta_c - \delta)/\sqrt{2\epsilon} < 1$, then $\Pi_\epsilon^{\text{gau}}(\delta_0; \delta_c, S) = \mathcal{O}(e^{-1/\epsilon})$

In fact, the expansion of the probability around the barrier becomes singular and one has to perform an expansion in terms of a layer variable $\eta = (\delta_c - \delta)/\sqrt{2\epsilon}$ (Knessl, 1986)

$$\Pi_\epsilon^{\text{gau}}(\delta_0; \delta_n, S_n) = C_\epsilon(\delta_0; \delta_n, S_n)u(\eta)$$

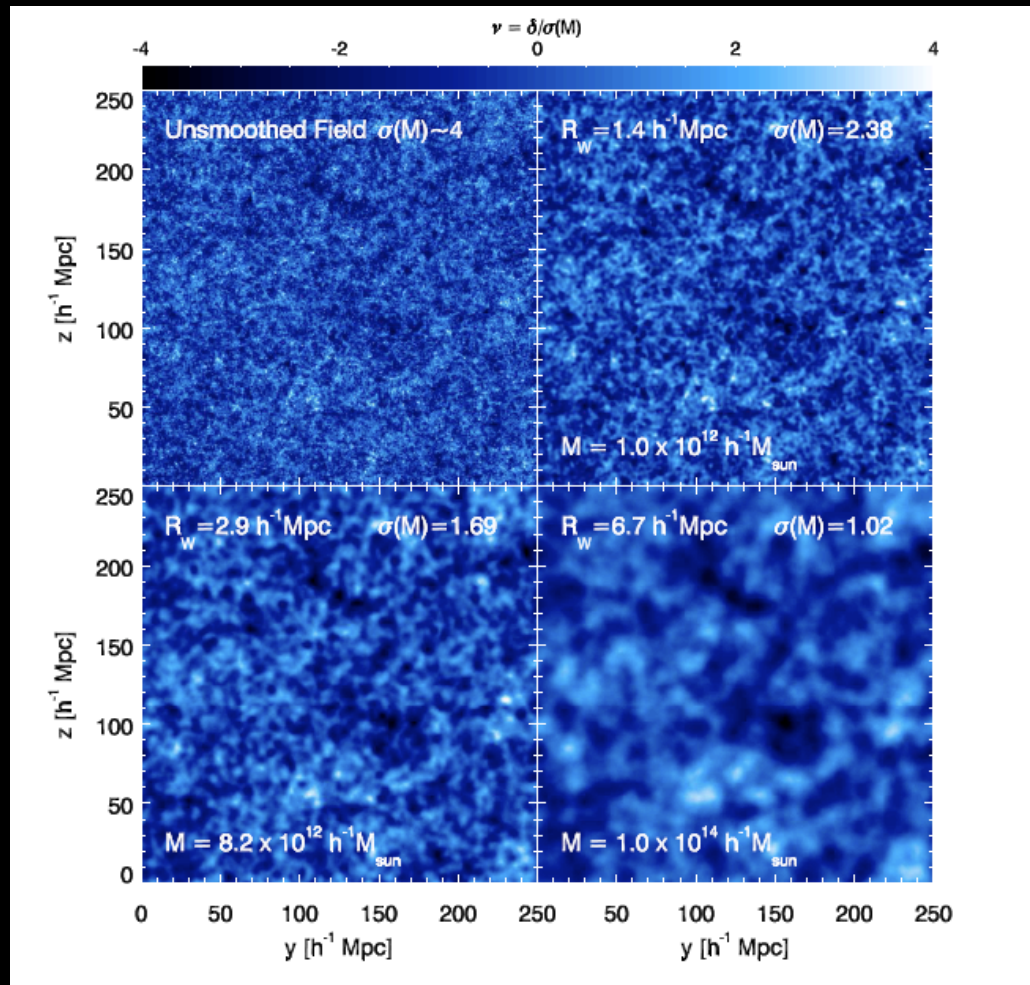
$$\lim_{\eta \rightarrow \infty} u(\eta) = 1$$

$$\lim_{\eta \rightarrow 0} \eta u(\eta) = \frac{1}{2}$$



$$\Pi_\epsilon^{\text{gau}}(\delta_0; \delta_c, S) = \sqrt{\frac{\epsilon}{\pi}} \frac{\delta_c - \delta_0}{S^{3/2}} e^{-(\delta_c - \delta_0)^2 / (2S)} + \mathcal{O}(\epsilon)$$

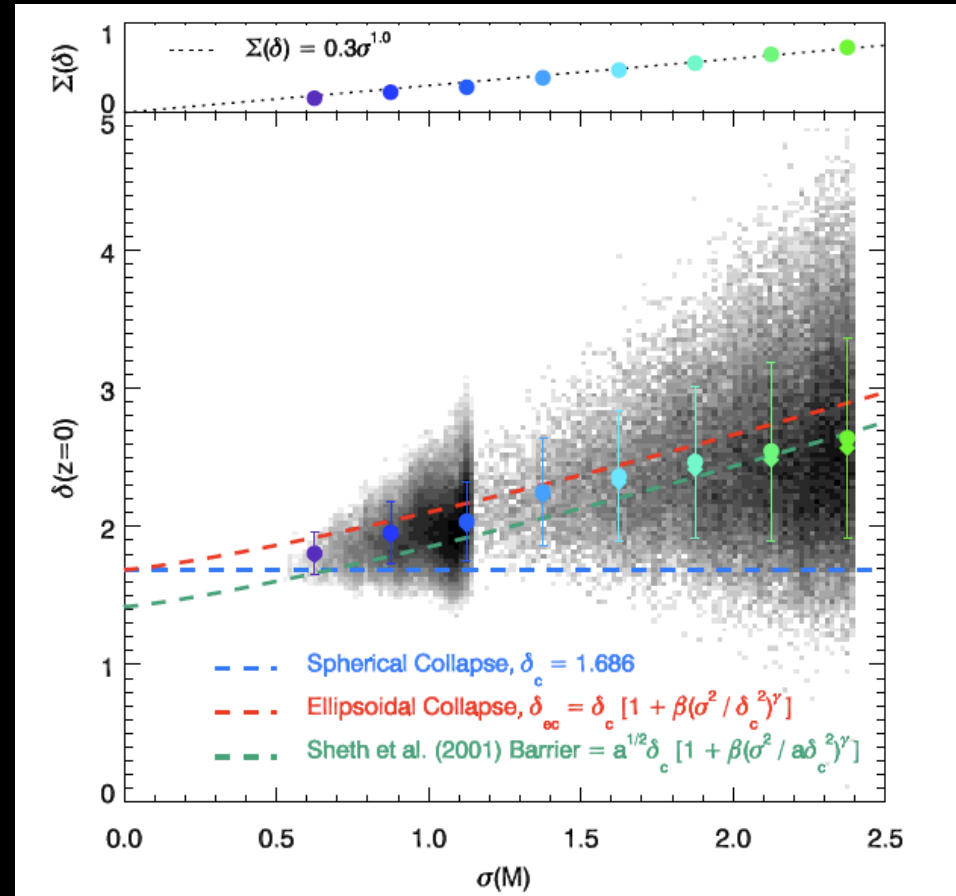
$$\Pi_\epsilon^{\text{gau}}(\delta_c; \delta_c, S) = \frac{\epsilon}{\sqrt{2\pi S^3}}$$



Robertson et al., 2009

For each halo identified, the center-of-mass of the halo particles is computed from their positions in the linear density field at $z \sim 100$ and use the window-smoothed field to compute the overdensity within the lagrangian radius R about this location. This overdensity is then linearly extrapolated to $z=0$

- The collapse barrier follows the ellipsoidal collapse model
- In the limit of large masses, the spherical collapse barrier is recovered
- The collapse barrier is not reduced to $0.84 \delta_c$



Robertson et al., 2009

The distribution of the smoothed linear overdensity is approximately log-normal in shape with a width

$$\Sigma_B \simeq 0.3 \sigma(M)$$

The scatter in the collapse barrier reflects the intrinsic scatter in the linear overdensity of collapsed regions introduced by the smoothing process

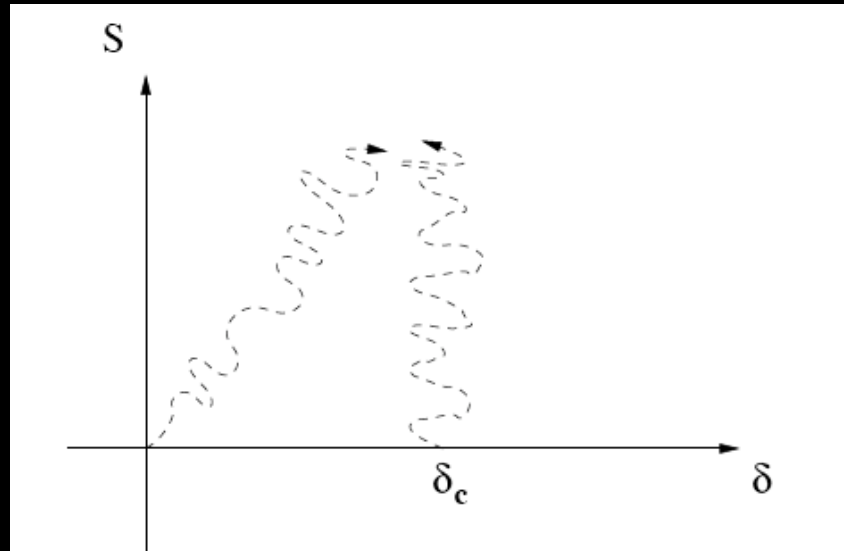
$$\begin{aligned}\langle (B - \langle B \rangle)^2 \rangle^{1/2} &= \left(e^{\Sigma_B^2} - 1 \right) \langle B \rangle \\ &\simeq \Sigma_B \langle B \rangle \\ &\simeq 0.3 \delta_c \sigma(M) = 0.3 \delta_c \sqrt{S}\end{aligned}$$

The collapse barrier moves stochastically with a diffusion coefficient

$$D_B \simeq (0.3 \delta_c)^2 \simeq 0.25$$

It encodes in an effective way the properties of the ellipsoidal collapse model (like the shear)
M. Maggiore, C. Porciani, R. Sheth and A.R., in prep.

The first-passage time problem becomes the well-known problem of the “diffusing cliff”



The first-passage time problem of two particles with diffusion coefficients $D = 1$ and $D_B = 0.25$ is mapped into a one-degree problem of a stochastic particle with effective coefficient

$$D_{\text{eff}} = 1 + D_B = 1.25$$

1. Non-markovian effects

2. Diffusing barrier

$$f(\sigma) = (1 - \tilde{\kappa}) \left(\frac{2a}{\pi} \right)^{1/2} \frac{\delta_c}{\sigma} e^{-a\delta_c^2/(2\sigma^2)}$$

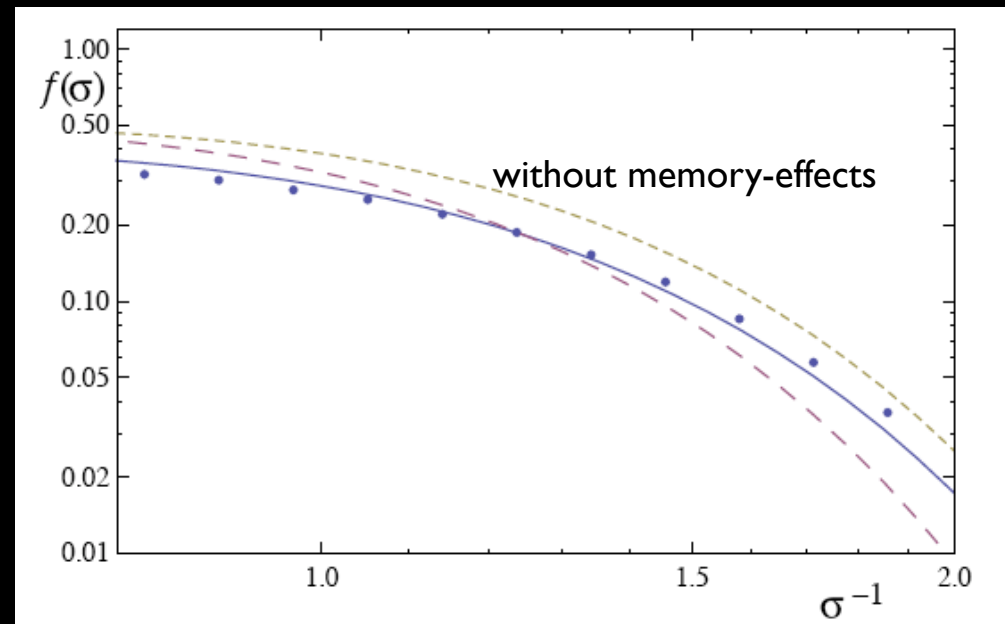
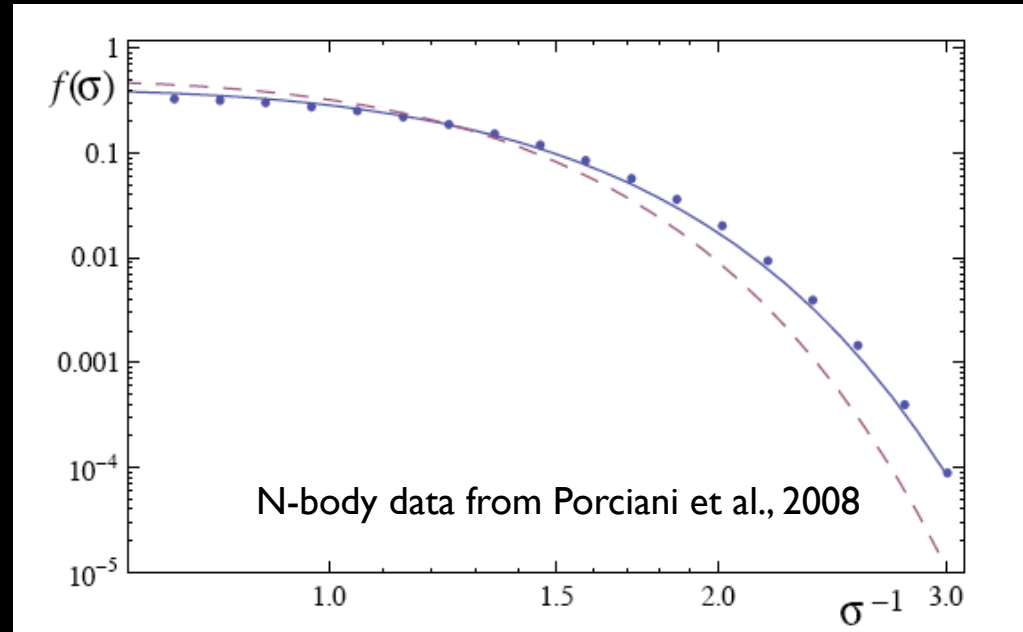
$$+ \frac{\tilde{\kappa} \delta_c \sqrt{a}}{\sigma \sqrt{2\pi}} \Gamma \left(0, \frac{a\delta_c^2}{2\sigma^2} \right)$$

$$a = \frac{1}{1 + D_B} \simeq 0.79$$

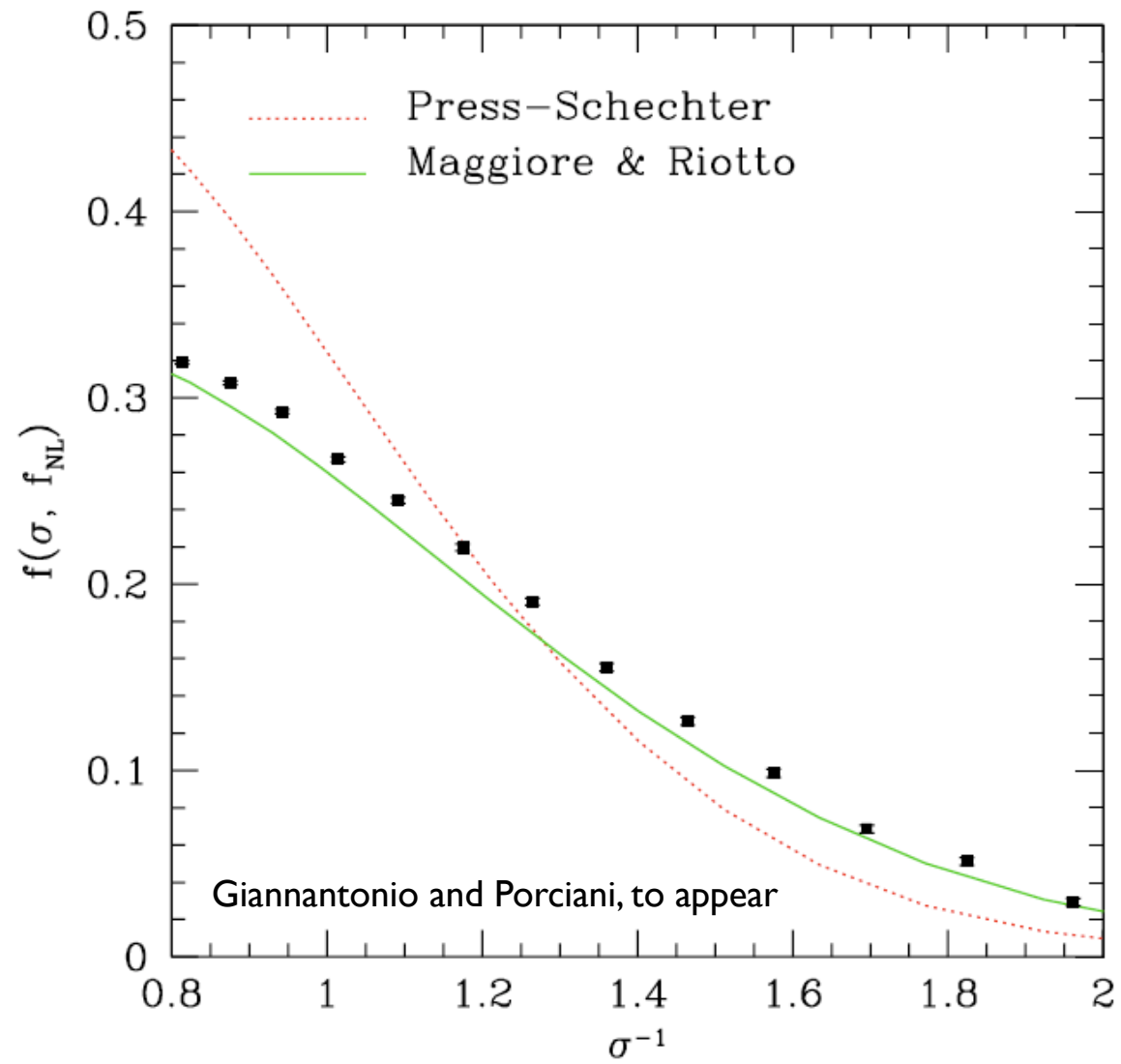
$$\tilde{\kappa} = \frac{\kappa}{1 + D_B}$$

The fact that the barrier fluctuates results in a larger effective diffusion coefficient, so it is easier for the smoothed density perturbation to reach the barrier. This is in fact due to the well-known phenomenon of "recurrence" of diffusion: in one-dimension two diffusing particles are certain to meet eventually.

No ad-hoc parameters



$$f_{\text{NL}} = 100$$



Conclusions & future prospects

- ✓ The halo mass function is a crucial quantity in the cosmological setting. At present, its knowledge is mainly due to N-body simulations. New data expected in the near future will test merging theory, bias theory and NG
- ✓ The excursion set method represents a powerful tool to calculate the dark matter halo mass function and can be used to describe the merging of haloes, to derive the level of bias, etc. However, analytical results have been obtained so far only for unrealistic filters which provide a markovian dynamics
- ✓ An approach has been developed from first principles to compute the halo mass function for any generic filter and for gaussian and non-Gaussian theories. The method efficiently deals with the non-markovian dynamics and memory effects
- ✓ A computation of the halo mass function has been provided which is in remarkable agreement with the N-body results. Two main effects concur to this agreement: 1) the memory effects and 2) the realization that the collapse barrier is diffusing
- ✓ These results may be applied to the merger theory, to the bias theory, NG and to the formation history where the non-markovian dynamics enters in a crucial way